UC More, Feel More, Do More:

Augmenting Engineering Mathematics Lectures and Tutorials with Visual, Tactile and Kinaesthetic Learning Aids

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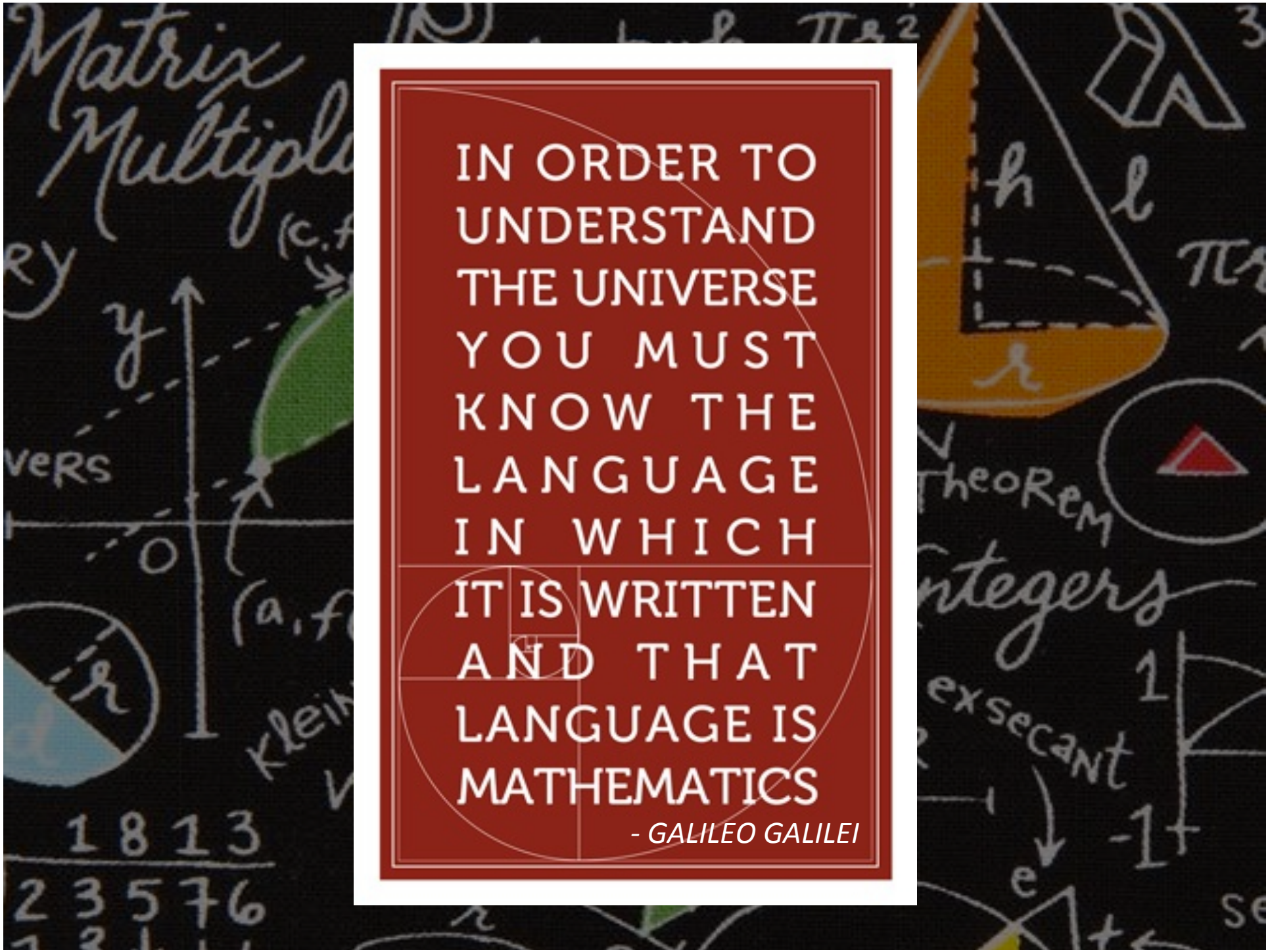
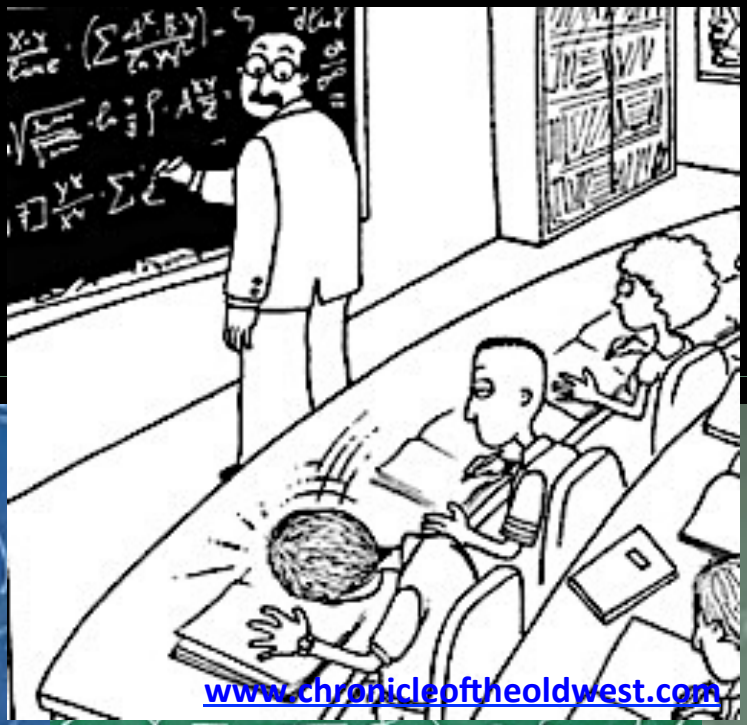
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND


IN ORDER TO UNDERSTAND THE UNIVERSE YOU MUST KNOW THE LANGUAGE IN WHICH IT IS WRITTEN AND THAT LANGUAGE IS MATHEMATICS
- GALILEO GALILEI

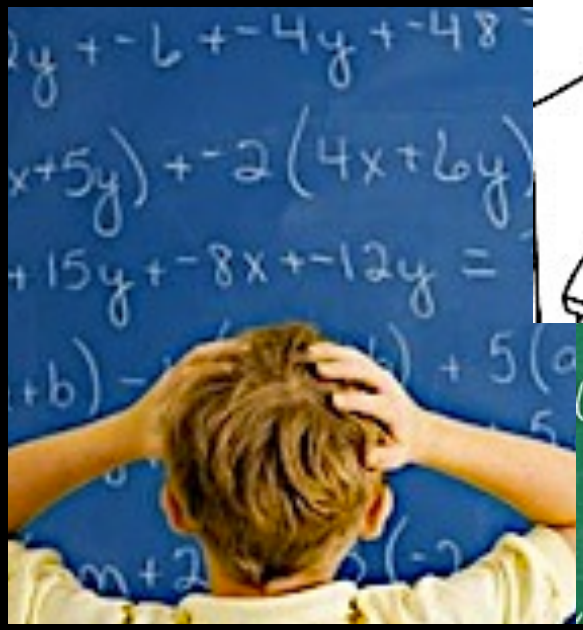
GOAL IN THE CLASSROOM

We need our students to
UNDERSTAND CONCEPTS CLEARLY,
Not just follow formulaic recipes

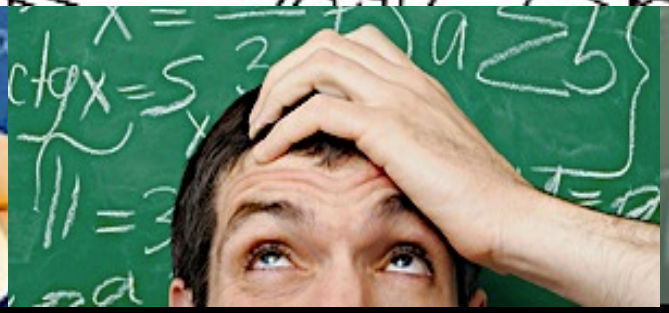
REALITY IN THE CLASSROOM



www.chronicleoftheoldwest.com



www.chemistryland.com



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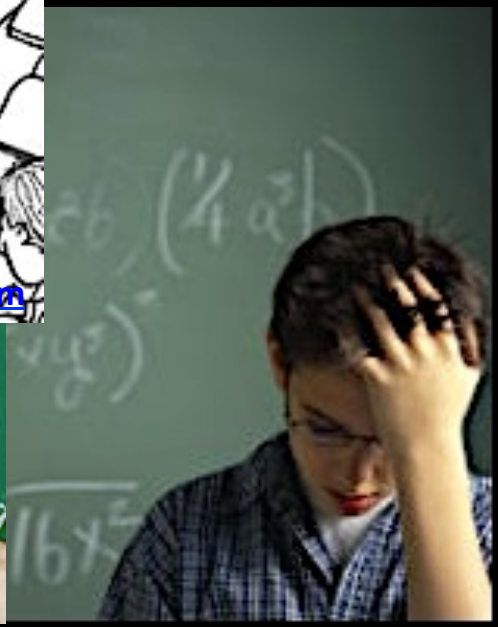


www.bobgriggs.com

THE PROBLEM

- We have a limited time
  to cover a comprehensive syllabus
  to a large number of students
- Mathematics is a difficult language!

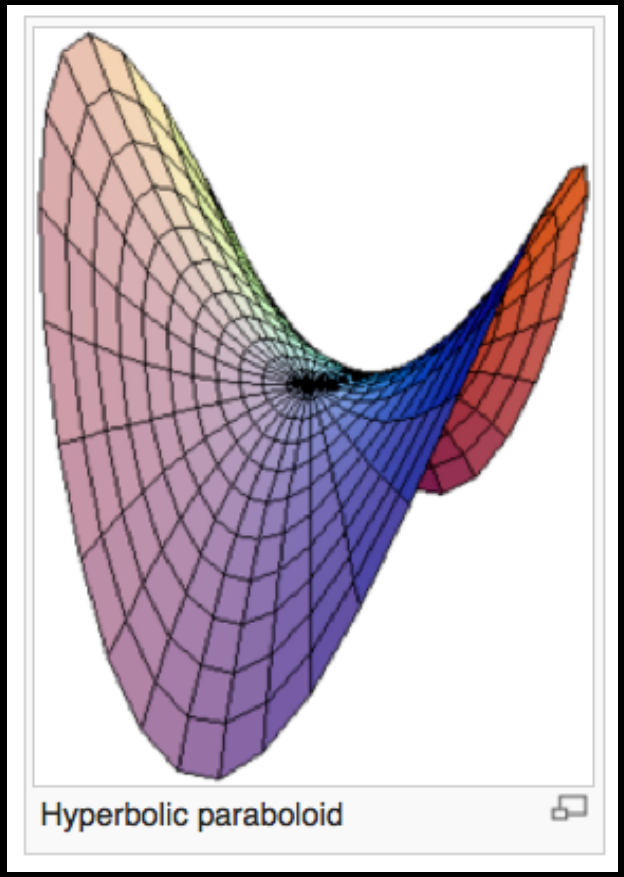
WE NEED TO COMMUNICATE BEYOND TRADITIONAL READ/WRITE METHODS BY USING MORE VISUAL, TACTILE & KINAESTHETIC STRATEGIES

Example: Hyperbolic Paraboloid

Traditional Read/Write: Formula

$$\frac{z}{c} = \frac{y^2}{b^2} - \frac{x^2}{a^2}.$$

Visual: Image



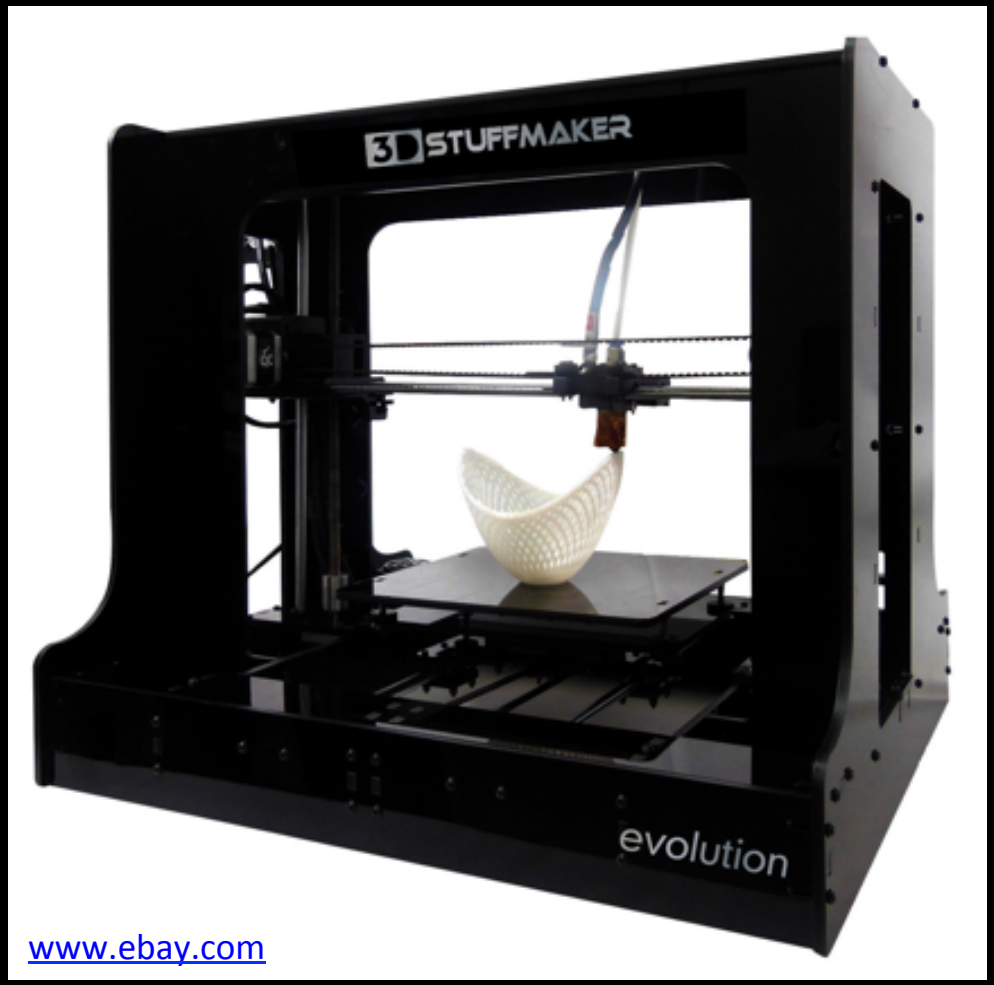
THE SOLUTION – *GOING A STEP FURTHER*

What if students could:

- SEE More
- FEEL More
- DO More

with the aid of Custom 3D-printed learning aids?

WHAT IS 3D PRINTING?



www.ebay.com

3D printers make manufacturing digital, personal and affordable

3D-PRINTED LEARNING AIDS

EXAMPLES OF CALCULUS SURFACES

Credit: Henry Segerman

youtube.com/watch?v=10KjAi5eA1Q

HOW TO PRINT CUSTOM 3D OBJECTS FOR TUTORIAL PROBLEMS: JUST 3 STEPS

MATHEMATICAL CONCEPT → COMPUTER MODEL → 3D PRINTED AID

(a) A hyperbolic paraboloid, showing slices through the surface in the x and y directions.

(b) A hyperbolic paraboloid, showing level curves.

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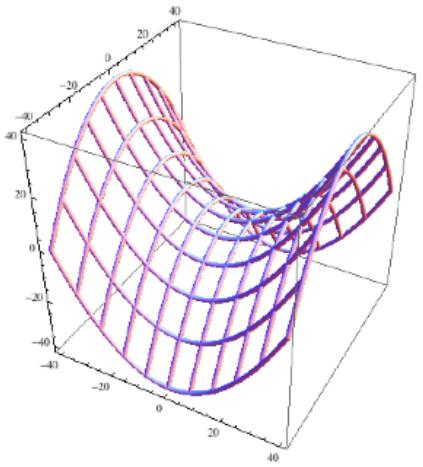
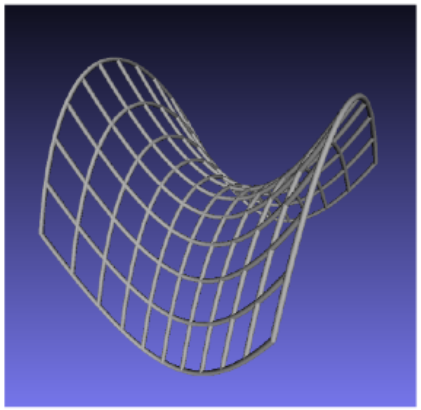
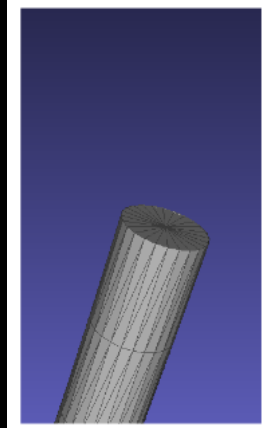


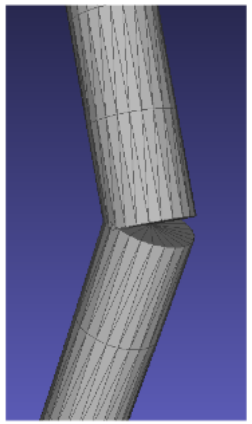
(a) Graphical output from Mathematica.



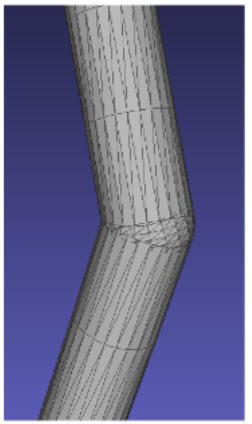
(b) The STL file as viewed in MeshLab.



(a) The "Tube" PlotStyle directive produces closed polygonal meshes.



(b) At a corner, these leave an ugly gap.



(c) This can be fixed by adding a sphere at the corner.

12 lines of MATHEMATICA Code

⬇

STL file for printing

```
1  f[u_, v_] := {u, v, u^2 - v^2};
2  scale = 40;
3  radius = 0.75;
4  numPoints = 24;
5  gridSteps = 10;
6  curvesU = Table[scale*f[u, i], {i, -1, 1, 2/gridSteps}];
7  curvesV = Table[scale*f[j, v], {j, -1, 1, 2/gridSteps}];
8  tubesU = ParametricPlot3D[curvesU, {u, -1, 1}, PlotStyle -> Tube[
       radius, PlotPoints -> numPoints], PlotRange -> All];
9  tubesV = ParametricPlot3D[curvesV, {v, -1, 1}, PlotStyle -> Tube[
       radius, PlotPoints -> numPoints], PlotRange -> All];
10 corners = Graphics3D[Table[Sphere[scale f[i, j], radius], {i, -1, 1,
       2}, {j, -1, 1, 2}], PlotPoints -> numPoints];
11 output = Show[tubesU, tubesV, corners]
12 Export["MathematicaParametricSurface.stl", output]
```

HOW TO PRINT CUSTOM 3D OBJECTS FOR TUTORIAL PROBLEMS: JUST 3 STEPS

MATHEMATICAL CONCEPT → COMPUTER MODEL → 3D PRINTED AID

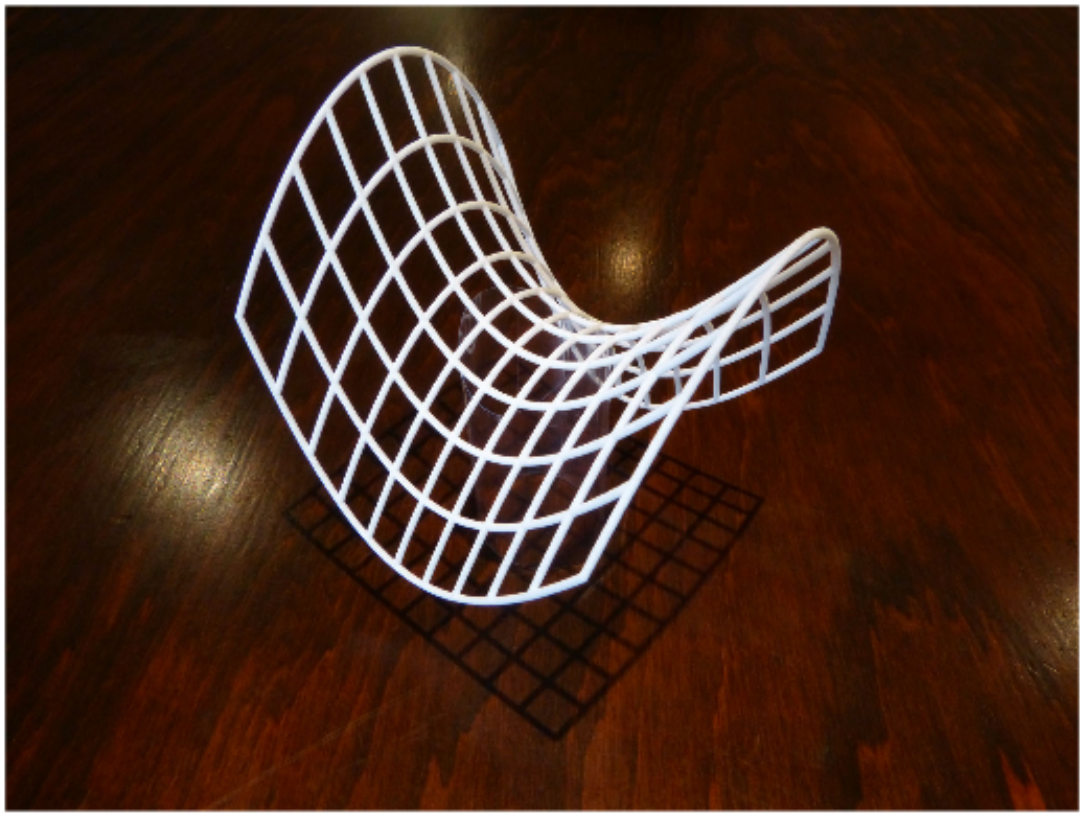


Figure 9: The 3D printed object.

3D-PRINTED LEARNING AIDS

Scaling to suit classroom and tutorial style learning

Credit: Henry Segerman

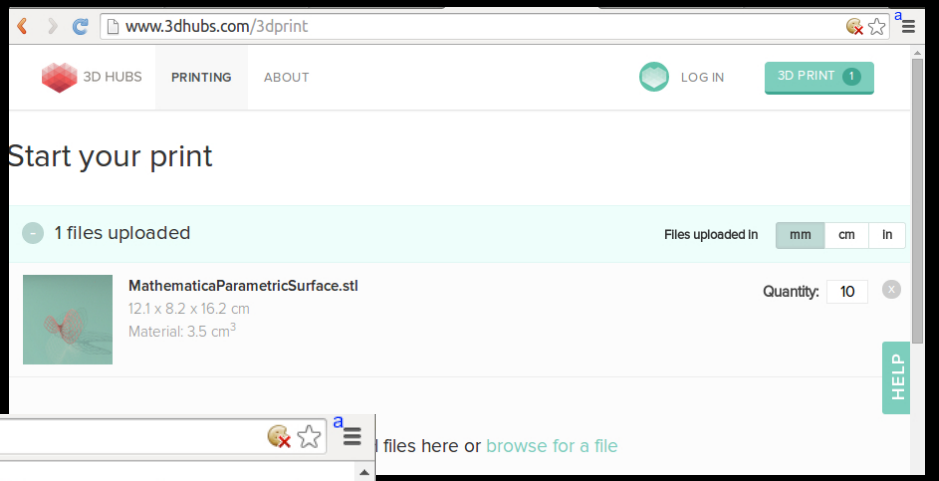
youtube.com/watch?v=LBiiOEiD3Yk

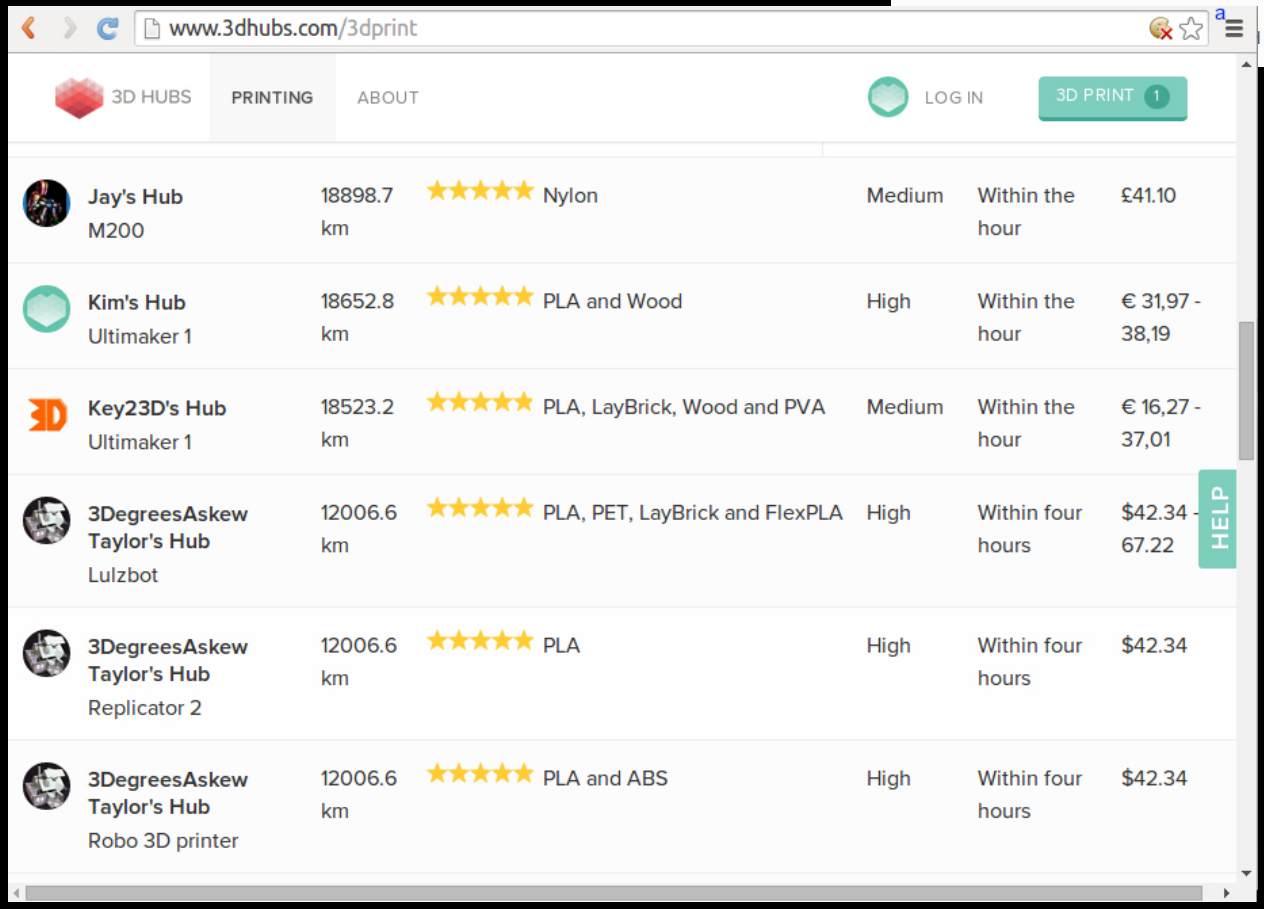
MORE EMTH CONCEPTS THAT CAN BE AUGMENTED WITH LEARNING AIDS

- 2D & 3D Integration
- Volumes of Rotation
- Line & Contour Integrals
- Polar & Spherical Coordinates
- Transformation of random vectors
- Tangent Planes
- Linear Independence

FEASIBILITY

- Use online services





Average Tutorial:

6 Problems

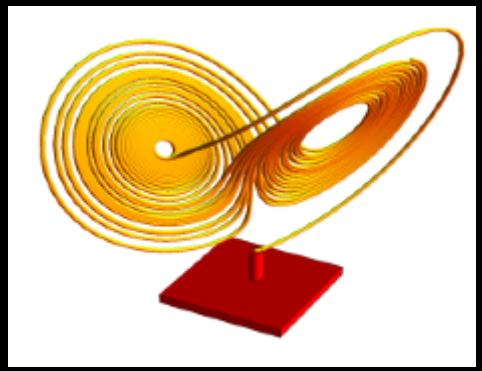
3 Parallel Sessions

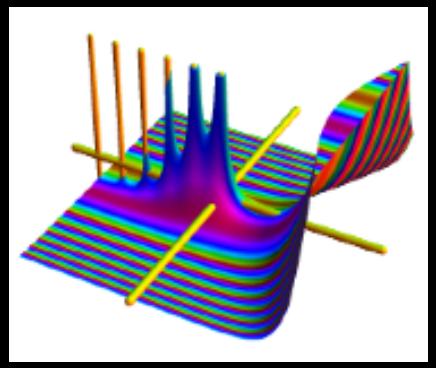
Will need 18 Objects

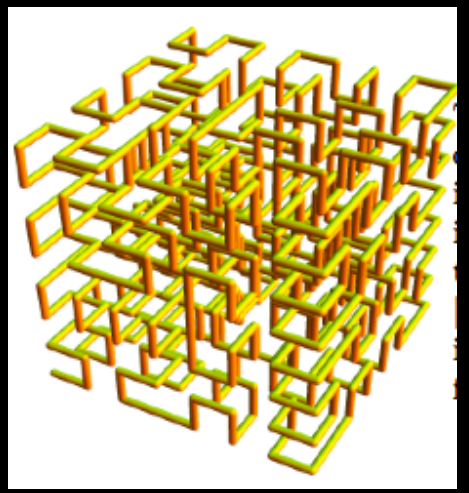
Cost for this example 72 USD

FEASIBILITY

- Buy a designated printer for the school
  - Cost effective in the long run
  - More applications for
    - Honours courses





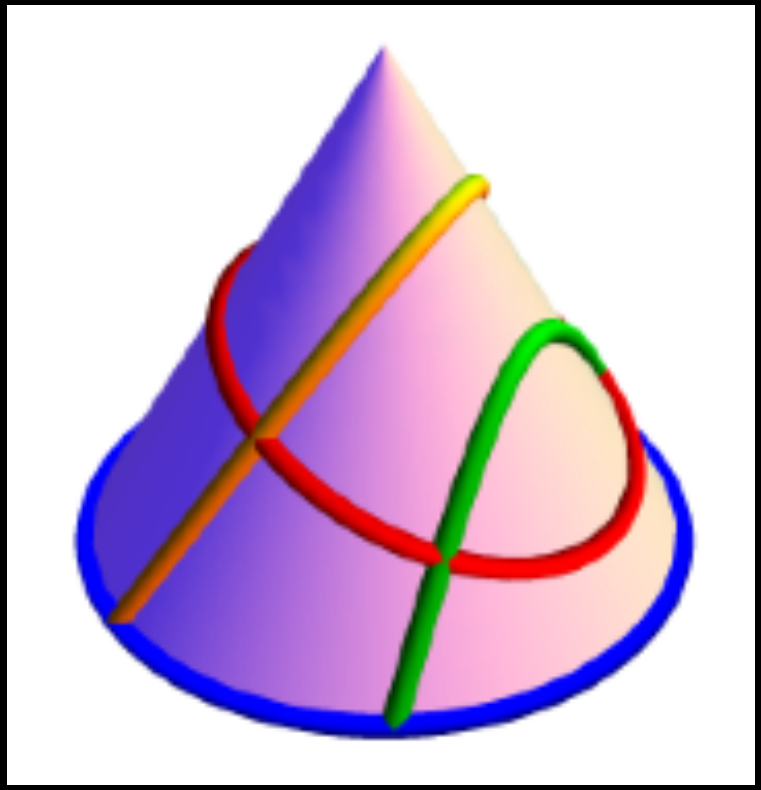


- Marketing/Outreach

MEASURING EFFICACY

- Solicit Direct Feedback from
  - Students on their learning experience in the Course Survey
  - Tutors on Student Engagement
- Compare student performance in examinations with historical records (SDB)

CONCLUSION



For conceptual clarity, there's nothing better than being able to see, feel and manipulate a mathematical object with one's bare hands!

REFERENCES

SEGERMAN H (2012) 3D Printing for Mathematical Visualisation.

KNILL O and SLAVKOVSKY E (2013) Illustrating Mathematics Using 3D Printers.

Thank you