Characterizing the Twitter networks of prominent politicians and hate groups in the USA

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http://lamastex.org/lmse/mep/, http://lamastex.org/preprints/2017HateIn2016USAElection.pdf

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Three Questions

- (Q1) Is Trump preferentially retweeted by various types of hate groups or their leadership relative to other politicians (i.e., Clinton, Sanders, Cruz, or Ryan) against the null random network model of *apathetic retweeting*?

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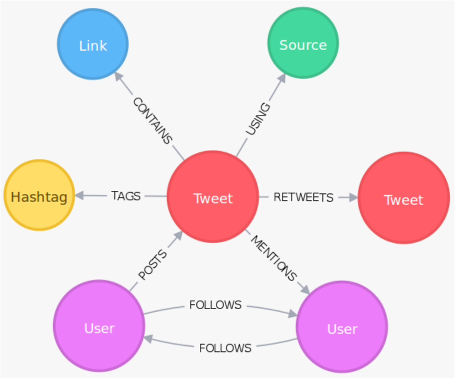
- (Q2) What frequency of unique users retweeted both a politician and a hate group or its leadership more than expected under the null model?
- (Q3) What is the joint distribution of the *degrees of separation* to each user from each of the five politicians and the eight most prolific hateful ideologies on Twitter, measured through the lengths of the most retweeted directed paths in the observed network?

Twitterverse

twitter is a micro-blogging service...

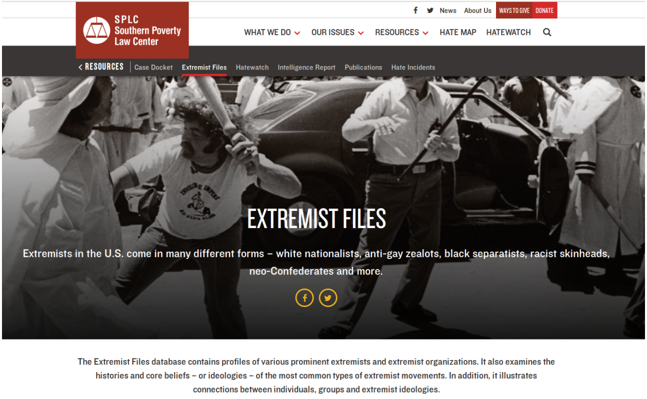
What is a tweet? retweet? reply-tweet, etc. (*status updates*)



Via public streams and REST APIs we collected ∼22M status updates related to 5 politicians and 52 hate groups (retrospective REST-based network augmentations).

Hateful Networks

US Hate Groups by SPLC https://www.splcenter.org/fighting-hate/extremist-files



- https://www.splcenter.org/fighting-hate/extremist-files/ideology
- https://www.splcenter.org/fighting-hate/extremist-files/individual
- https://www.splcenter.org/fighting-hate/extremist-files/groups

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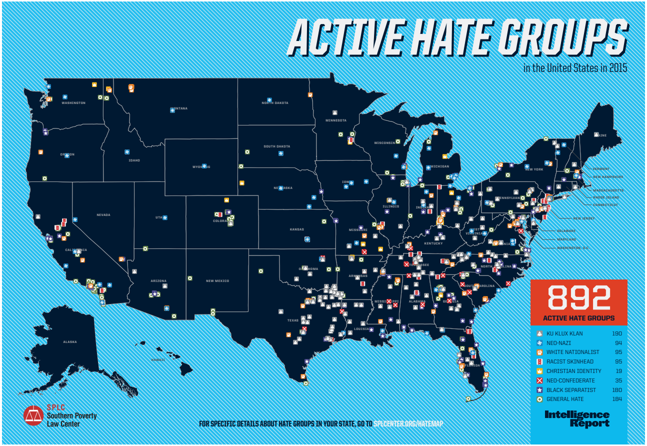

https://www.splcenter.org/hate-map

Hateful Networks

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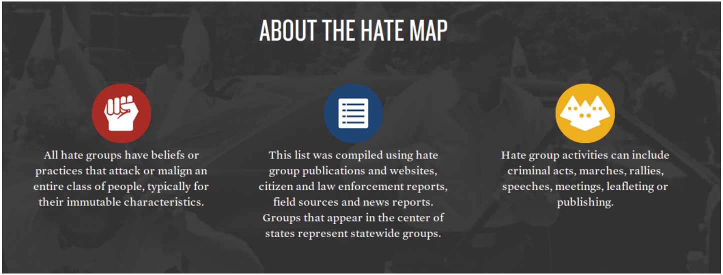

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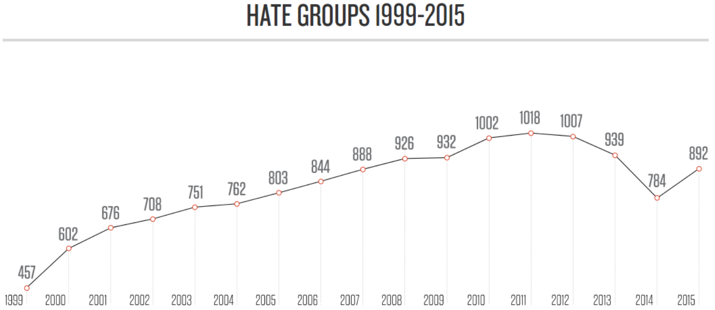

https://www.splcenter.org/hate-map

Hateful Networks

18 US Hateful Ideologies by SPLC

https://www.splcenter.org/fighting-hate/extremist-files

Alternative Right

The Alternative Right, commonly known as the Alt-Right, is a set of far-right ideologies, groups and individuals whose core belief is that “white identity” is under attack by multicultural forces using “political correctness” and “social justice” to undermine white people and “their” civilization...

Anti-Immigrant

Anti-immigrant hate groups are the most extreme of the hundreds of nativist and vigilante groups that have proliferated since the late 1990s, when anti-immigration xenophobia began to rise to levels not seen in the United States since the 1920s.

Anti-LGBT

Opposition to equal rights for LGBT people has been a central theme of Christian Right organizing and fundraising for the past three decades – a period that parallels the fundamentalist movement's rise to political power.

https://www.splcenter.org/fighting-hate/extremist-files/ideology

Hateful Networks

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Anti-Muslim

Anti-Muslim hate groups are a relatively new phenomenon in the United States, most of them appearing in the aftermath of the World Trade Center terrorist attacks on Sept. 11, 2001. Earlier anti-Muslim groups tended to be religious in orientation and disputed Islam's status as a respectable religion.

Antigovernment Movement

The antigovernment movement has experienced a resurgence, growing quickly since 2008, when President Obama was elected to office. Factors fueling the antigovernment movement in recent years include changing demographics driven by immigration, the struggling economy and the election of the first...

Black Separatist

Black separatists typically oppose integration and racial intermarriage, and they want separate institutions -- or even a separate nation -- for blacks. Most forms of black separatism are strongly anti-white and anti-Semitic, and a number of religious versions assert that blacks are the Biblical "...

https://www.splcenter.org/fighting-hate/extremist-files/ideology

Hateful Networks

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Christian Identity

Christian Identity is a unique anti-Semitic and racist theology that rose to a position of commanding influence on the racist right in the 1980s. "Christian" in name only, the movement's relationship with evangelicals and fundamentalists has generally been hostile due to the latter's belief that...

General Hate

These groups espouse a variety of rather unique hateful doctrines and beliefs that are not easily categorized. Many of the groups are vendors that sell a miscellany of hate materials from several different sectors of the white supremacist movement.

Holocaust Denial

Deniers of the Holocaust, the systematic murder of around 6 million Jews in World War II, either deny that such a genocide took place or minimize its extent. These groups (and individuals) often cloak themselves in the sober language of serious scholarship, call themselves "historical revisionists...

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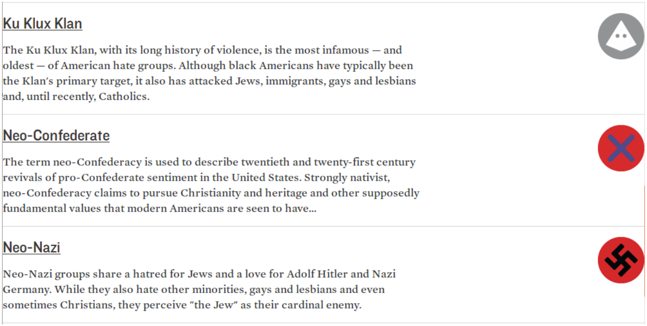

Ku Klux Klan

The Ku Klux Klan, with its long history of violence, is the most infamous — and oldest — of American hate groups. Although black Americans have typically been the Klan's primary target, it also has attacked Jews, immigrants, gays and lesbians and, until recently, Catholics.

Neo-Confederate

The term neo-Confederacy is used to describe twentieth and twenty-first century revivals of pro-Confederate sentiment in the United States. Strongly nativist, neo-Confederacy claims to pursue Christianity and heritage and other supposedly fundamental values that modern Americans are seen to have...

Neo-Nazi

Neo-Nazi groups share a hatred for Jews and a love for Adolf Hitler and Nazi Germany. While they also hate other minorities, gays and lesbians and even sometimes Christians, they perceive "the Jew" as their cardinal enemy.

https://www.splcenter.org/fighting-hate/extremist-files/ideology

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Phineas Priesthood

The Phineas Priesthood is not an actual organization; it has no leaders, meetings, or any other institutional apparatus.

Racist Music

Racist music groups are typically white power music labels that record, publish and distribute racist music in a variety of genres.

Racist Skinhead

Racist Skinheads form a particularly violent element of the white supremacist movement, and have often been referred to as the "shock troops" of the hoped-for revolution. The classic Skinhead look is a shaved head, black Doc Martens boots, jeans with suspenders and an array of typically racist...

`https://www.splcenter.org/fighting-hate/extremist-files/ideology`

Hateful Networks

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Radical Traditional Catholicism

"Radical traditionalist" Catholics, who may make up the largest single group of serious anti-Semites in America, subscribe to an ideology that is rejected by the Vatican and some 70 million mainstream American Catholics.

Sovereign Citizens Movement

The strange subculture of the sovereign citizens movement, whose adherents hold truly bizarre, complex antigovernment beliefs, has been growing at a fast pace since the late 2000s. Sovereigns believe that they get to decide which laws to obey and which to ignore, and they don't think they should...

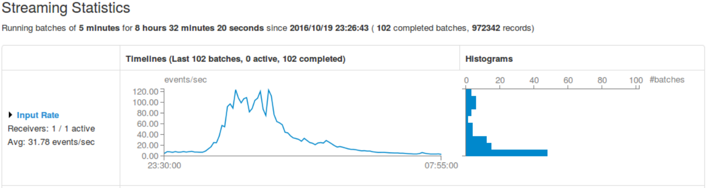
White Nationalist

White nationalist groups espouse white supremacist or white separatist ideologies, often focusing on the alleged inferiority of nonwhites. Groups listed in a variety of other categories - Ku Klux Klan, neo-Confederate, neo-Nazi, racist skinhead, and Christian Identity - could also be fairly...

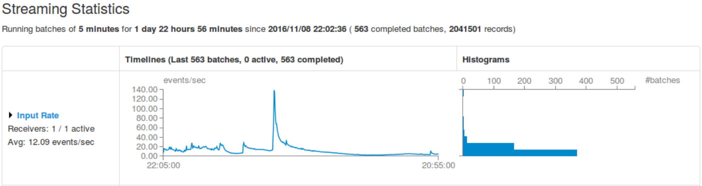
https://www.splcenter.org/fighting-hate/extremist-files/ideology

US Presidential Election 2016 - Twitter Streams

Twitter Data — 3rd US Presidential Debate



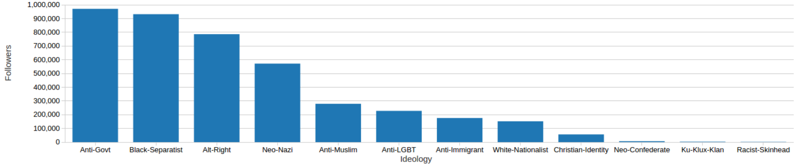
Twitter Data — Last 2 Days Around the End of Election



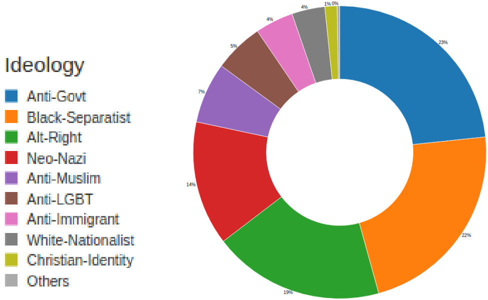
- public streams of `@realDonaldTrump`, `@HillaryClinton`, `@BernieSanders`, `@tedcruz`, `SpeakerRyan` and 52 splc-defined hategroups of their leadership
- collected data includes all mentions, replies, retweets, etc of these twitter accounts of interest for about 9 weeks around the 2016 US Presidential Election

12 SPLC-defined hateful ideologies

– **only 78% of hategroups identified by SPLC were active in Twitter**



Ideology	Followers
Anti-Govt	970769
Black-Separatist	931736
Alt-Right	786327
Neo-Nazi	571772
Anti-Muslim	279122
Anti-LGBT	227636
Anti-Immigrant	175441
White-Nationalist	151711
Christian-Identity	56191
Neo-Confederate	6628
Ku-Klux-Klan	3070
Racist-Skinhead	1826



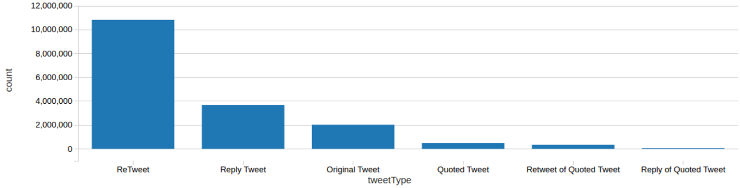
5 prominent Politicians in the USA

Retweet Network statistics of the five politicial accounts

Politician	in-degree	in-nbhd	out-degree	out-nbhd
Donald Trump	40	12	5,952,257	958,262
Hillary Clinton	225	121	2,774,111	943,995
Bernie Sanders	107	62	762,209	356,718
Paul Ryan	769	158	68,973	28,902
Ted Cruz	322	189	49,479	27,663

Dataset overview

Data collected around the 2016 US Presidential Election



- data = 2.7M tweets, 13.7M retweets, 22M status updates

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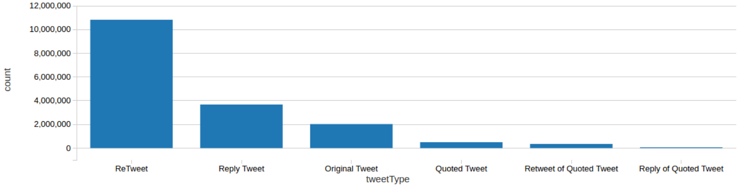

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- 4.4M distinct retweet-pairs: (original-Tweeter, Retweeter)

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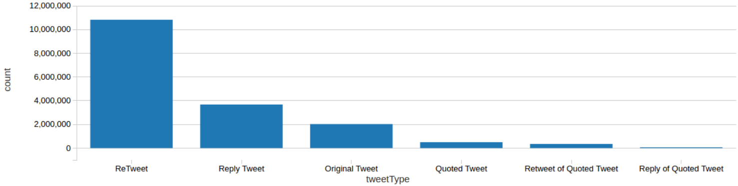


- data = 2.7M tweets, 13.7M retweets, 22M status updates
- 4.4M distinct retweet-pairs: (original-Tweeter, Retweeter)
- 2.5M unique users

Retweet Network — (3% sample $\#\mathbb{V} = 1205$, $\#\mathbb{E} = 29856$)

Trump-Clinton Retweet Network — a few samples

CPostUserSN	OPostUserSNinRT	OPostUserSNinQT	favouritesCount	followersCount	friendsCount	isVerified	isGeoEnabled	CurrentTweet
georgefayner	realDonaldTrump	null	137811	1466	953	false	true	RT @realDonaldTrump: China is cooking up conspiracy theories that the Olympics are rigged. http://t.co /0ah0hBJt They don't understand why…
KevinCormier10	realDonaldTrump	null	16164	505	367	false	true	RT @realDonaldTrump: EXCLUSIVE: FBI Agents Say Comey 'Stood In The Way' Of Clinton Email Investigation: https://t.co /6n63fHVvNo
thuerta	realDonaldTrump	null	13081	128	345	false	true	RT @realDonaldTrump: 'Trump rally disrupter was once on Clinton's payroll' https://t.co /75oLLuD4SI
tanladyvolfan	HillaryClinton	null	6316	101	200	false	true	RT @HillaryClinton: Our progress is on the ballot. Tolerance is on the ballot. Democracy is on the ballot. Make a plan to vote:…

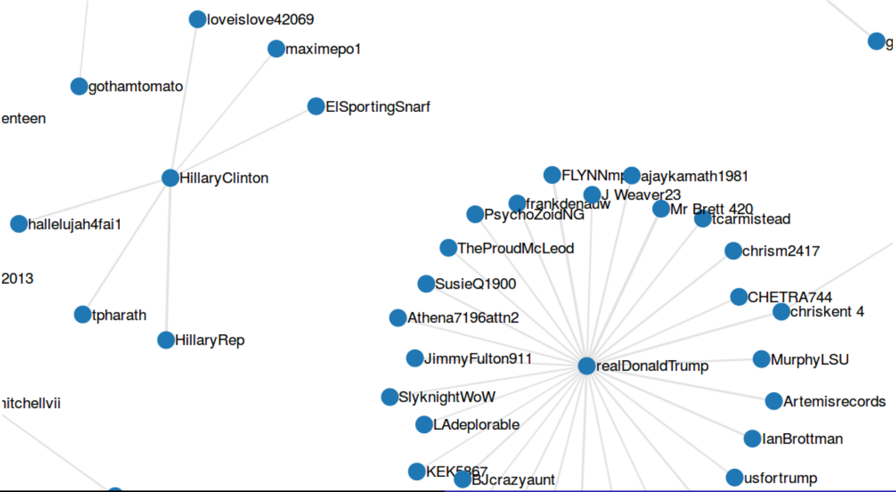
Retweet Network — (3% sample $\#\mathbb{V} = 1205, \#\mathbb{E} = 29856$)

Trump-Clinton Retweet Network weighted by Retweet counts

userCreatedAtDate	daysSinceUserCreated	OPostUserSNinRT	CPostUserSN	max(favouritesCount)	max(followersCount)	max(friendsCount)	ReTweetCount
2011-12-13T19:10:28.000+0000	1781	realDonaldTrump	Mr_Brett_420	3294	78	194	190
2016-04-30T00:13:34.000+0000	181	HillaryClinton	HillaryRep	4196	2168	4984	158
2011-03-22T13:09:23.000+0000	2047	realDonaldTrump	FLYNNmpc	1653	48	75	146
2014-08-25T17:02:46.000+0000	795	realDonaldTrump	mikenyr2499	17427	183	155	132
2009-04-26T07:07:03.000+0000	2742	yottapoint	gcommking	5876	797	1826	120
2014-06-20T21:37:39.000+0000	861	BUILDseriesNYC	suzannebuzz	38604	1706	486	112
2009-05-28T15:51:31.000+0000	2710	realDonaldTrump	chriskent_4	838	254	85	112
2009-03-08T12:59:18.000+0000	2791	realDonaldTrump	Artemisrecords	2000	2777	5000	112
2012-09-25T15:09:37.000+0000	1494	realDonaldTrump	IanBrottman	1	89	151	107
2011-03-31T00:54:09.000+0000	2038	realDonaldTrump	frankdenauw	43	45	18	102
2016-07-17T21:30:47.000+0000	103	HillaryClinton	loveislove42069	3818	108	398	98
2015-09-01T18:52:08.000+0000	423	realDonaldTrump	BJcrazyaunt	1064	1296	1433	95
2011-12-24T03:52:02.000+0000	1770	HillaryClinton	tpharath	703	38	183	91
2015-03-08T23:47:05.000+0000	600	HillaryClinton	hallelujah4fai1	16766	227	270	88
2014-06-30T16:44:10.000+0000	851	realDonaldTrump	ajaykamath1981	3309	2667	3010	88
2012-04-29T21:49:38.000+0000	1643	realDonaldTrump	MurphyLSU	65	28	47	84
2010-08-05T16:02:11.000+0000	2276	realDonaldTrump	sdpubs	23674	123	34	83
2011-07-24T19:55:57.000+0000	1923	realDonaldTrump	chrism2417	3012	182	1112	81
2016-02-03T23:58:01.000+0000	268	realDonaldTrump	SusieQ1900	6797	386	415	81

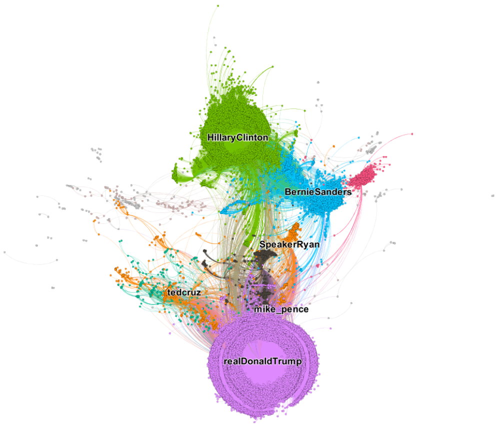
Retweet Network — (3% sample $\#\mathbb{V} = 1205$, $\#\mathbb{E} = 29856$)

Trump-Clinton Retweet Ideological Network



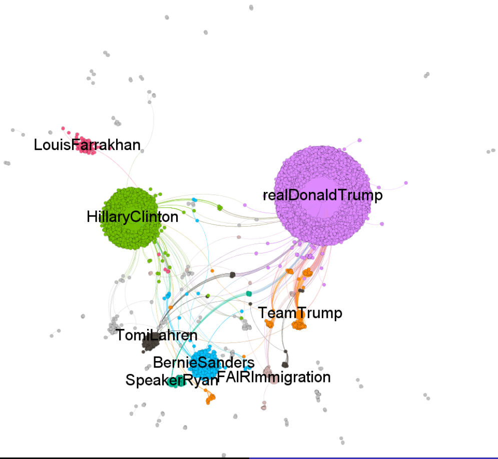
Strong Community Structure – samples of retweet networks

The 3rd US Presidential Debate 22K Retweet Network



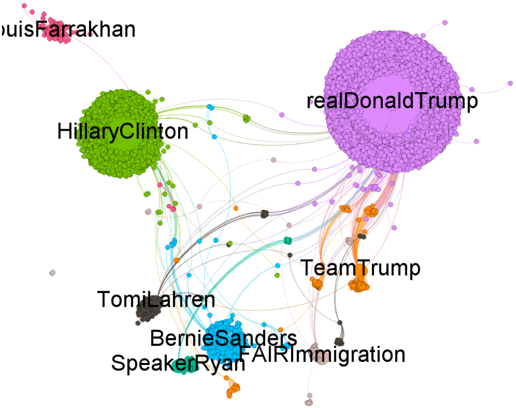
Strong Community Structure – samples of retweet networks

5% random sampled retweet networks for October 19 2016



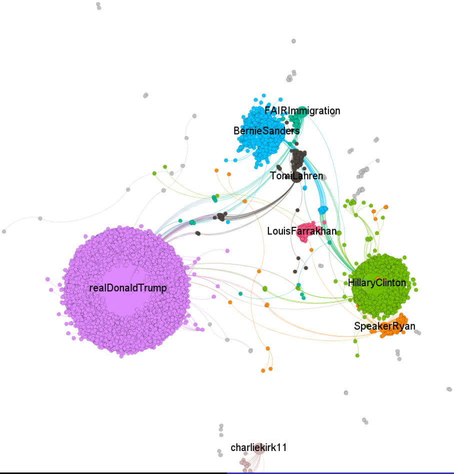
Strong Community Structure – samples of retweet networks

5% random sampled retweet networks for October 19 2016 – top 10



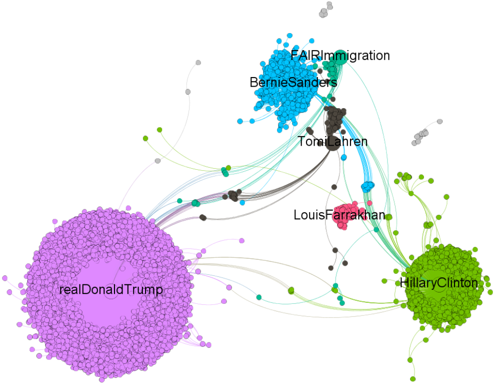
Strong Community Structure – samples of retweet networks

5% random sampled retweet networks for October 24 2016



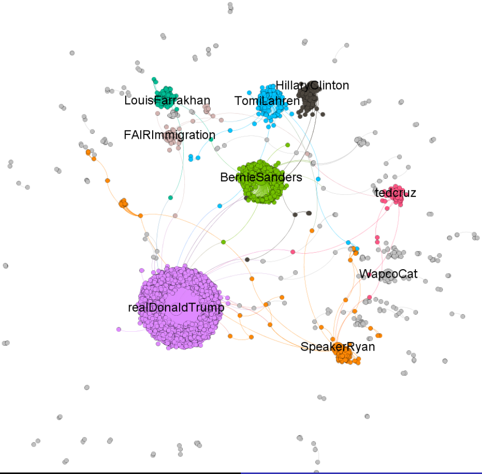
Strong Community Structure – samples of retweet networks

5% random sampled retweet networks for October 24 2016 – top 6



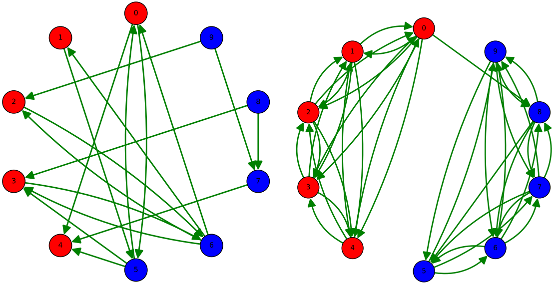
Strong Community Structure – samples of retweet networks

5% random sampled retweet networks for November 15 2016



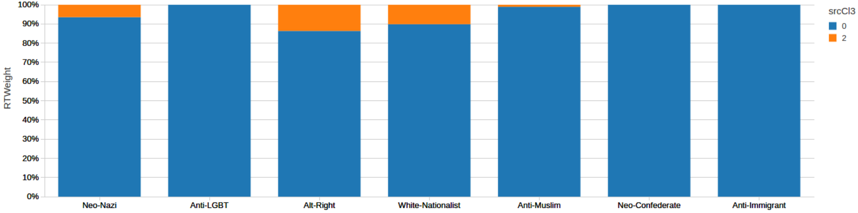
Models for Ideological Network Dynamics

- If arc $a_{i,j} = 1$ then we say i ideologically concurs with j



- Just two retweet networks out of $4,722,366,482,869,645,213,696$ for 9 individuals!
- We want indegree and outdegree conditioned random networks to preserve observed heterogeneity
- This is the classical *random directed configuration model* – H_0: *apathetic retweet network*
- NEED: distributed computing using Apache Spark (fastest growing Apache project)

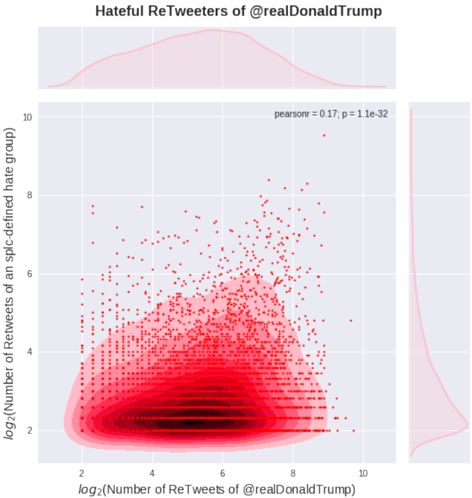
7 SPLC-defined hateful ideologies Retweet Proportions



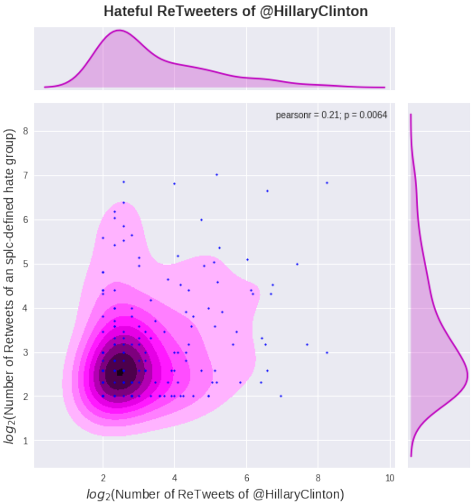
0 = Trump's cluster and 2 = Clinton's cluster

A significant proportion of retweets by leaders of seven extremist ideologies have original tweets in Trump's ideological cluster.

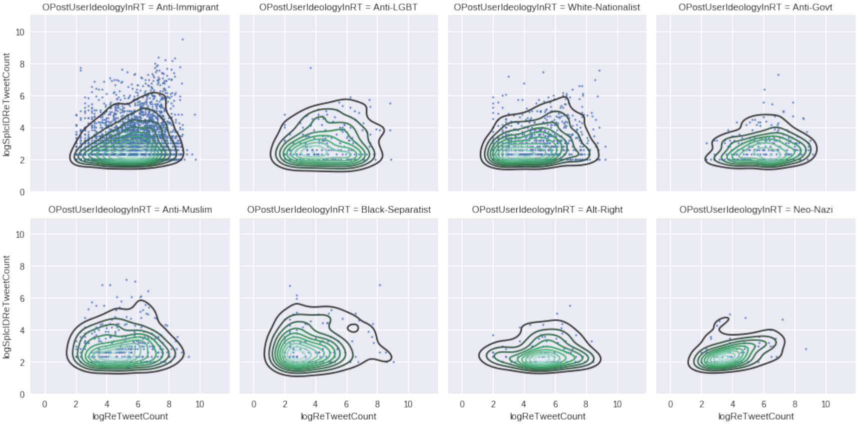
Trump's Hateful Retweeters



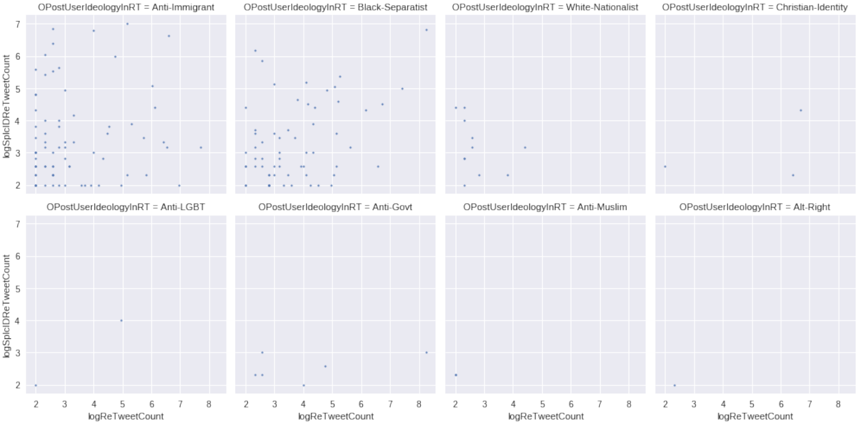
Clinton's Hateful Retweeters



Trump's Hateful Retweeters By Ideology



Clinton's Hateful Retweeters By Ideology



Chi-square tests – do NOT account for network heterogeneity

Ideology	Donald J Trump	Hillary R Clinton	Chi-Square Statistic	R^2
Alt-Right	1127 (90.7%)	116 (9.3%)	$X^2 = 822.30$, $p < .0001$	$R^2 = 0.662$
Anti-Government	1455 (89.5%)	171 (10.5%)	$X^2 = 1013.93$, $p < .0001$	$R^2 = 0.623$
Anti-Immigrant	15019 (88.6%)	1926 (11.4%)	$X^2 = 10116.65$, $p < .0001$	$R^2 = 0.597$
Anti-LGBT	1621 (88.6%)	209 (11.4%)	$X^2 = 1089.48$, $p < .0001$	$R^2 = 0.595$
Anti-Muslim	2293 (90.8%)	233 (9.2%)	$X^2 = 1679.97$, $p < .0001$	$R^2 = 0.665$
Black-Separatist	1279 (54.9%)	1049 (45.1%)	$X^2 = 22.72$, $p < .01$	$R^2 = 0.009$
Neo-Nazi	1039 (90.7%)	106 (9.3%)	$X^2 = 760.25$, $p < .0001$	$R^2 = 0.664$
White-Nationalist	5103 (89.2%)	616 (10.8%)	$X^2 = 3520.40$, $p < .0001$	$R^2 = 0.616$
Total	28992 (86.5%)	4509 (13.5%)	$X^2 = 18006.72$, $p < .0001$	$R^2 = 0.540$

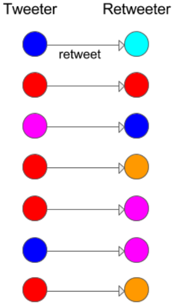
Chi-square tests – do NOT account for network heterogeneity

Restricting to retweeters who retweet at least 4 times

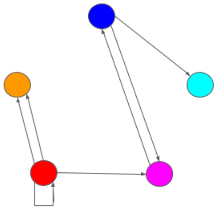
Ideology	Donald J Trump	Hillary R Clinton	Chi-Square Statistic	R^2
Alt-Right	936 (98.7%)	12 (1.3%)	$X^2 = 900.61$, $p < .0001$	$R^2 = 0.950$
Anti-Government	1388 (98.4%)	23 (1.6%)	$X^2 = 1320.50$, $p < .0001$	$R^2 = 0.936$
Anti-Immigrant	12618 (96.6%)	442 (3.4%)	$X^2 = 11351.84$, $p < .0001$	$R^2 = 0.869$
Anti-LGBT	1110 (96.0%)	46 (4.0%)	$X^2 = 979.32$, $p < .0001$	$R^2 = 0.847$
Anti-Muslim	1866 (98.8%)	22 (1.2%)	$X^2 = 1801.03$, $p < .0001$	$R^2 = 0.954$
Black-Separatist	494 (62.5%)	296 (37.5%)	$X^2 = 49.63$, $p < .001$	$R^2 = 0.062$
Neo-Nazi	692 (99.4%)	4 (0.6%)	$X^2 = 680.09$, $p < .0001$	$R^2 = 0.977$
White-Nationalist	3751 (98.0%)	76 (2.0%)	$X^2 = 3529.04$, $p < .0001$	$R^2 = 0.922$
Total	22855 (96.1%)	921 (3.9%)	$X^2 = 20234.71$, $p < .0001$	$R^2 = 0.851$

Cut-Permute-Rewire – distributed, scalable, and fault-tolerant sampler - in pictures

Directed Retweet Edges as Two Columns



Multi-edged Self-looped Retweet Network



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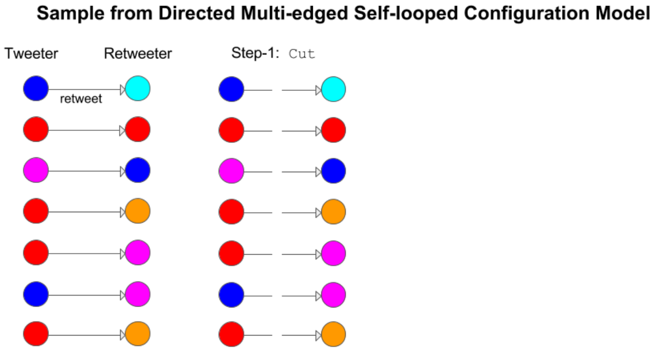

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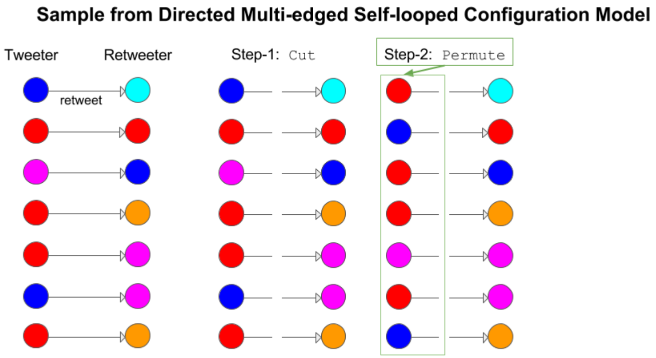
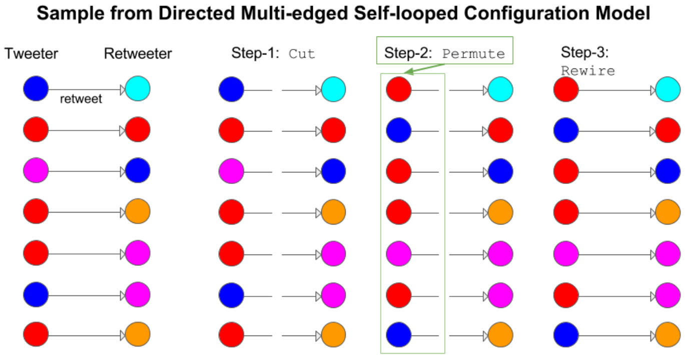


This random permutation of row #'s of observed outbound half-edges is: $(1, 2, 3, 4, 5, 6, 7) \mapsto (7, 6, 5, 4, 3, 2, 1)$

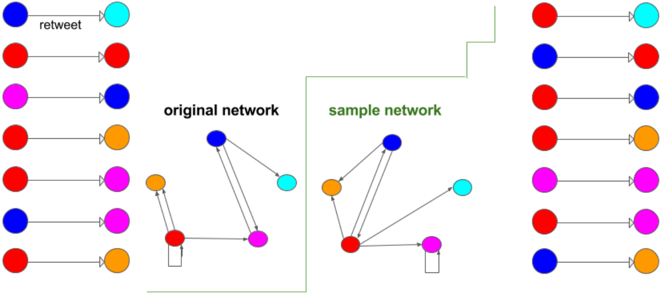
Cut-Permute-Rewire – distributed, scalable, and fault-tolerant sampler - in pictures



Cut-Permute-Rewire – distributed, scalable, and fault-tolerant sampler - in pictures

Sample from Directed Multi-edged Self-looped Configuration Model

Thus, we can sample from the scalable fault-tolerant `Cut-Permute-Rewire` algorithm



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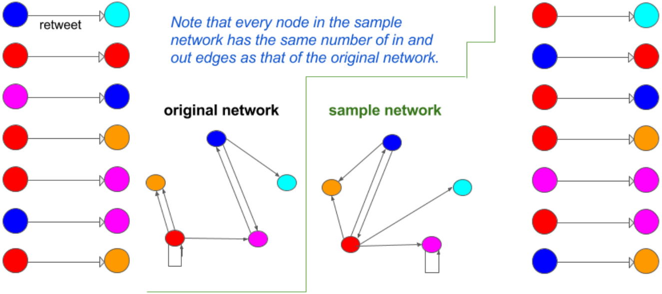

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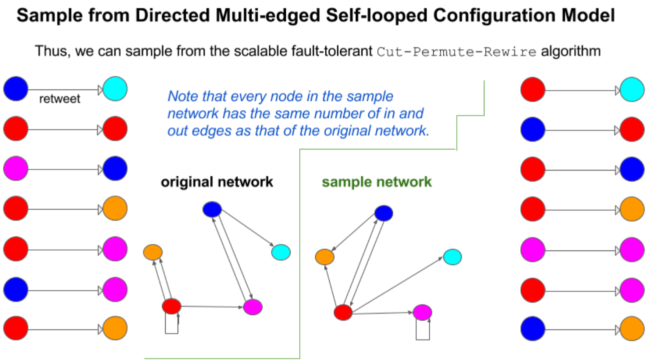

Question: What is the probability of the sample network?

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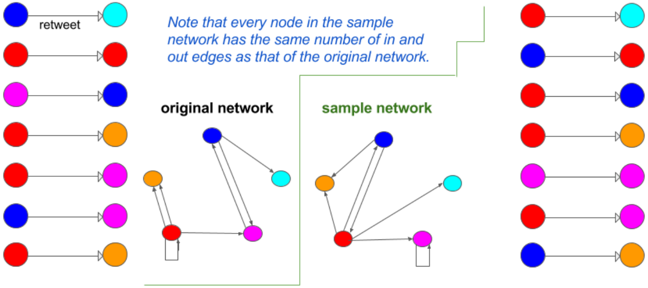

Question: What is the probability of the sample network?

Answer: $1/\#\text{edges}! = 1/\#\text{retweets}! = 1/7!$

$= 1/(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) = 1/(42 \times 60 \times 2) = 1/(252 \times 2) = 1/5040$

Cut-Permute-Rewire – distributed, scalable, and fault-tolerant sampler – in English

CUTPERMUTEANDREWIRE generates sample networks from the *directed multi-edged self-looped random configuration model* (Newman, Strogatz and Watts, 2001):

- *cutting* the directed edges representing the retweets in our observed retweet network into out-bound and in-bound half edges,

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Interpret the independent and identical samples as those from the null model H_0 as the *apathetic retweet model*

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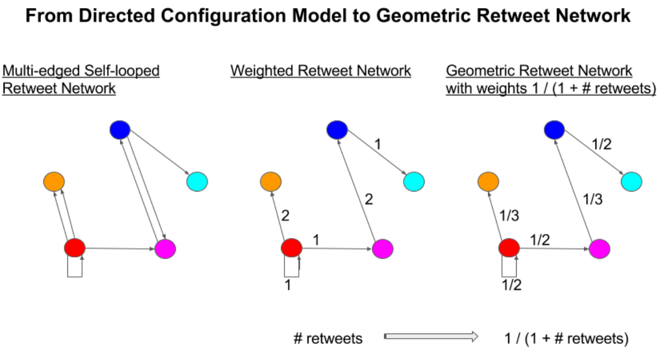
Interpret the independent and identical samples as those from the null model H_0 as the *apathetic retweet model* – "this is not reality folks!"

An Empirical Geomertic Retweet Nework & Most Retweeted Directed Paths — is born when distributed

Djkstra meets Poisson whose Expectation is Random Exponential with observed number of retweets as its mean parameter



interpretation: In a Geometric Retweet Network, the shortest directed path from *a* to *b* is the "most retweeted

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Empirical Geometric Retweet Network + distributed multiple-sources shortest paths vertex programs
$\longrightarrow$ The "Where Am I?" Operator in Evolving *Population Ideological Trees and Forests*

- choose a set I of "influential" nodes of interest (choice is informed by the empirical out-neighborhoods and out-degrees typically)
- $I \mapsto$ most retweeted path lenths to several subsets of I
- $\mapsto$ *Population Ideological Tree of Interest*.
- $\mapsto$ *Population Ideological Forest of Interest* (due to multi-component rwtewwet networks).

(Q1) Relative frequency of retweets by any one of the hate groups or their leadership for any original tweet made by one of the politicians

Null distribution of the test statistic under the apathetic retweet network model.

Table 2. Relative frequency of retweets by any one of the hate groups or their leadership for any original tweet made by one of the politicians

Politician, observed test statistic: marginal interval for the region of acceptance at 0.001 significance level				
Donald Trump	Hillary Clinton	Bernie Sanders	Paul Ryan	Ted Cruz
0.987 : (0.6008,0.6013)	0 : (0.2708,0.2709)	0 : (0.0677,0.0682)	0: (0.00411,0.00413)	0.0131 : (0.0024,0.0028)

(Q2) Number of unique users who retweeted a politician and a hate group at least five times each

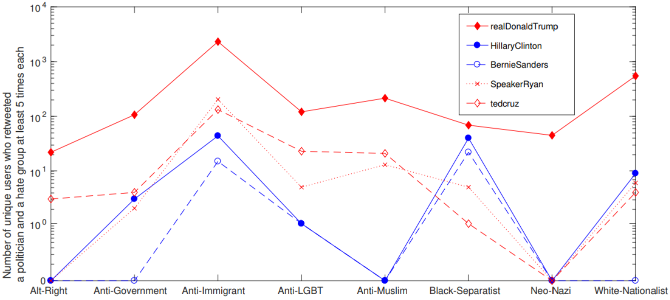


Fig. 1. Number of unique users who retweeted a politician and a hate group at least five times each (Note: The y-axis is in log-scale in powers of 10).

(Q2) Number of unique users who retweeted a politician and a hate group at least five times each

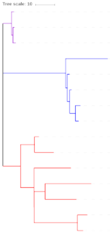
Null distribution of the test statistic under the apathetic retweet network model.

Table 3. Observed frequency of distinct users who retweeted a politician and a leader within a hate group at least 5 times each

Ideology	Politician				
	Donald Trump	Hillary Clinton	Bernie Sanders	Paul Ryan	Ted Cruz
	observed test statistic: marginal interval for the region of acceptance at 0.001 significance level				
Anti-Government	*107 : (0, 1)	3 : (0, 3)	0 : (0, 1)	*2 : (0,1)	*4 : (0, 1)
Anti-Immigrant	*2314 : (375, 498)	°44 : (373, 492)	°15 : (369, 485)	*204 : (47, 95)	*133 : (18, 54)
Anti-LGBT	*121 : (0, 4)	1 : (0, 4)	1 : (0, 4)	*5 : (0, 3)	*23 : (0, 3)
Anti-Muslim	*215 : (0, 3)	0 : (0, 3)	0 : (0, 3)	*13 : (0, 3)	*21 : (0, 3)
Neo-Nazi	*45 : (0, 1)	0 : (0, 1)	0 : (0, 1)	0: (0, 1)	0: (0, 1)
White-Nationalist	*548 : (0, 12)	9 : (0, 10)	0 : (0, 10)	6: (0, 8)	4: (0, 7)
Black-Separatist	°69 : (653, 811)	°40 : (649, 808)	°22 : (645, 801)	°5 : (72, 128)	°1 : (28, 66)
Alternative-Right	*22 : (0, 0)	0 : (0, 0)	0 : (0, 0)	0 : (0, 0)	*3 : (0, 0)

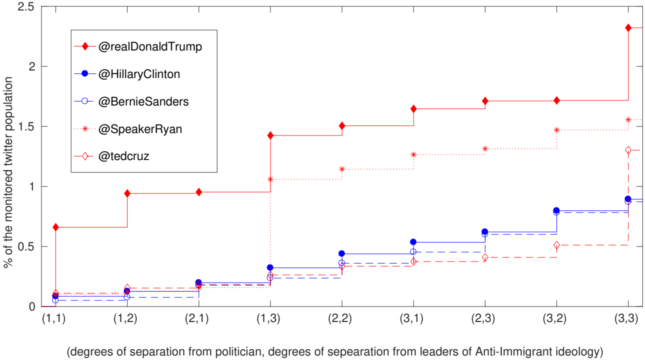
(Q3) Population ideological Tree & Degrees of Separation

Table 4. The top 15 groups of users according to their profiles of most retweeted path-lengths from the five politicians (DT = @realDonaldTrump, HC = @HillaryCinton, BS = @BernieSanders, PR = @SpeakerRyan, TC = @tedcruz) and eight hateful ideologies (AI = Anti-Immigrant, AM = Anti-Muslim, WN = White-Nationalist, AL = Anti-LGBT, AG = Anti-Govt, NN= Neo-Nazi, BIS=Black-Separatist, AR=Alt-Right) given by their id, frequency, percentage of population and their classification given by the ideological tree with leaf nodes as the ids.

ideological tree	id	frequency	percentage	Politician					Hate Group							
			of population	DT	HC	BS	PR	TC	AI	AM	WN	AL	AG	NN	BIS	AR
	05	42853	02.005	1	1	2	4	4	5	5	7	6	4	7	7	7
	07	11481	00.537	1	2	1	4	4	5	5	7	6	4	7	7	7
	12	5868	00.274	1	1	1	4	4	5	5	7	6	4	7	7	7
	11	5972	00.279	4	2	3	5	7	8	8	9	9	7	10	10	10
	02	791286	37.016	3	1	2	4	6	7	7	8	8	6	9	9	9
	04	74126	03.468	3	1	1	4	6	7	7	8	8	6	9	9	9
	03	232093	10.857	3	2	1	6	6	7	7	9	8	6	9	9	9
	13	5173	00.242	3	1	1	6	6	7	7	8	8	6	9	9	9
	01	811586	37.965	1	4	7	4	4	5	5	7	6	4	7	7	7
	15	3892	00.182	1	4	7	1	4	5	5	7	3	4	7	7	7
	14	4011	00.188	1	4	7	4	4	1	5	3	5	4	5	7	7
	08	10460	00.489	3	5	9	1	3	3	3	5	3	6	7	9	9
	09	8069	00.377	3	3	3	3	1	4	3	3	3	6	5	6	9
	06	29997	01.403	2	3	3	3	3	5	3	3	5	5	5	3	3
	10	6257	00.293	1	3	3	4	4	5	3	3	5	4	5	3	3

Zooming into the Joint Degrees of Sepration From each Politican and Hateful Ideology

Cumulative % of the monitored population who are within a given in-degree of separation from a politician and a hateful Ideology.



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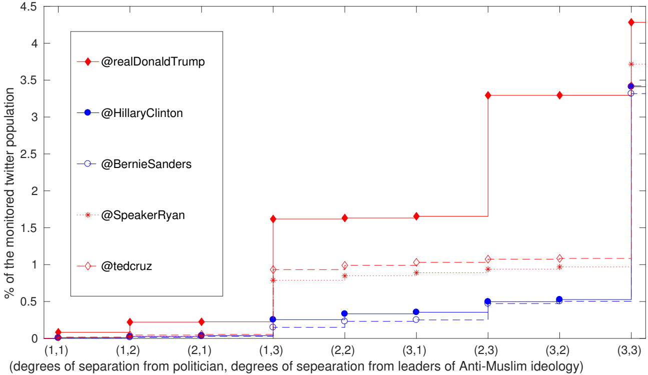

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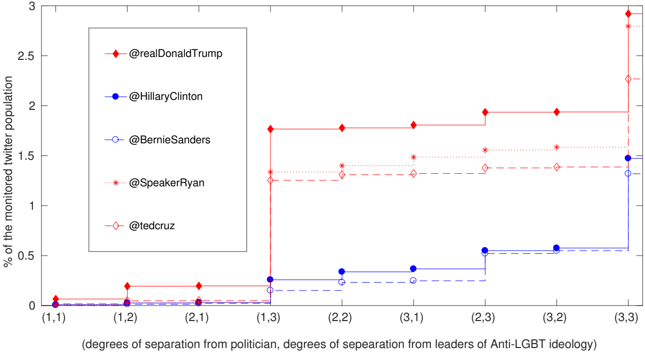

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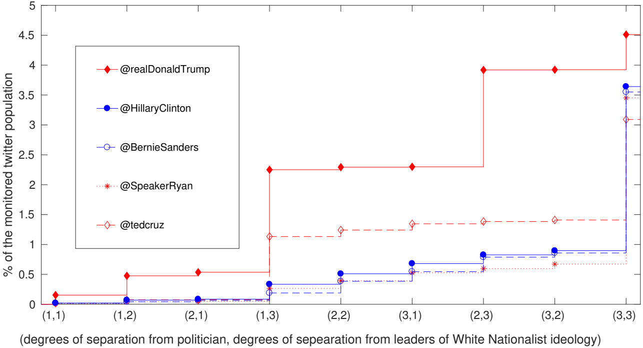

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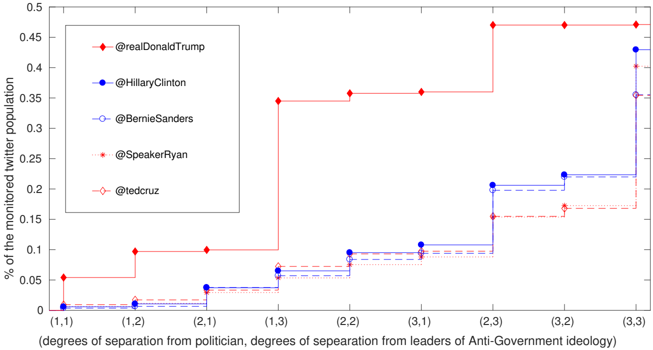

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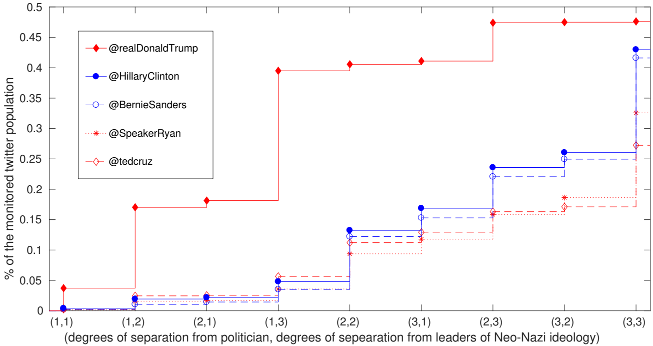

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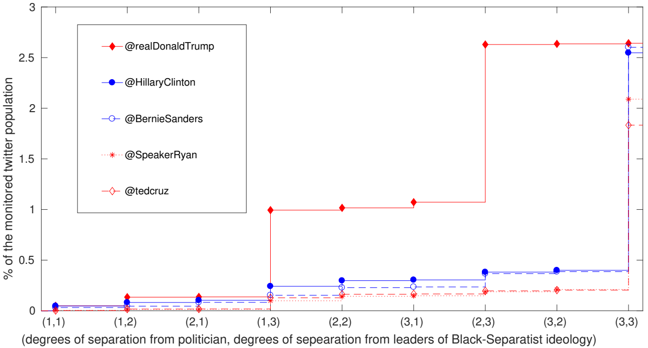

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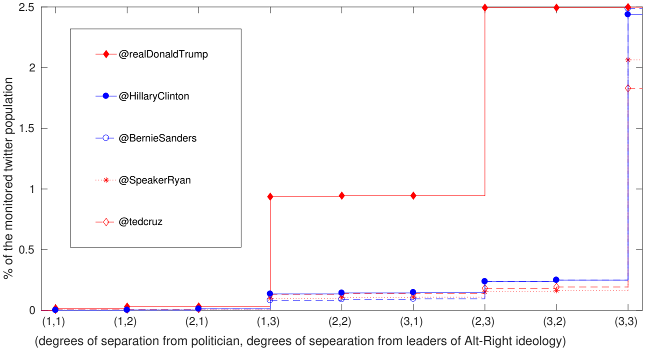

Significance Statement

During the 2016 US presidential election, there was significant debate on whether Donald Trump's campaign was fuelled by hate and bigotry toward minority groups. We analyzed nearly 22 million communication events on Twitter to better understand the networks of retweeters of American hate groups and five key American politicians during the late stages of the election (Donald Trump, Hillary Clinton, Bernie Sanders, Ted Cruz, and Paul Ryan). Our data reveals that Twitter users linked to various American hate groups including Anti-Government, Anti-Immigrant, Anti-LGBT, Anti-Muslim, Neo-Nazi and White-Nationalist were more strongly linked to Trump over any other politician.

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On a seemingly highly hopeful note about the "American people": Only a small fraction of those within 3 degrees of separation from @realDonalTrump during the 9 week period are also within 3 degrees of separation from any hateful ideology!

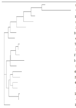
Generalizable Interactive Streaming-REST Design

A 10 Day Design for 2017 UK Election (post-Brexit)

- **Influencers of Interest**
  - **politicians:** Jeremy Corbyn (JC), Theresa May (TM), Angela Rayner (AR, Labour), Boris Johnson (BJ, Conservative)
  - **journalists and bloggers:** the Independent (In), the Daily Telegraph (Te), Robert Peston (RP, journalist and author), Piers Morgan (P, journalist, tv-personality), Keven Maguire (KM, journalist), Owen Jones (OJ, left, the guardian), Paul Mason (PM, left-wing journalist (the guardian etc.)), Louise Mench (LM, previously conservative mp now blogger), and Guido Fawkes (GF, right/liberal).

- **Population Ideological Tree** →

Classification Tree

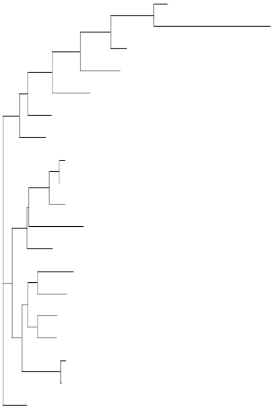


		Politicians				News/Bloggers								
Id	%	JC	TM	AR	BJ	In	Te	RP	P	KM	OJ	PM	LM	GF
9	1.4	5	5	6	6	5	6	5	6	6	5	6	6	6
7	1.6	'∞'	1	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'
10	1.4	5	6	5	5	5	5	5	5	5	4	5	5	6
8	1.5	4	6	4	6	6	1	4	4	5	6	4	5	5
3	3.3	4	6	6	4	1	4	4	4	6	3	4	6	3
189	0.1	3	1	4	5	4	3	5	3	5	3	4	5	3
6	1.8	4	5	3	5	4	3	3	3	3	3	5	4	4
73	0.1	3	1	3	3	3	2	3	1	3	2	3	4	3
2	7.6	3	2	3	3	3	2	3	1	3	2	3	4	3
173	0.1	3	2	3	4	1	2	3	1	3	2	4	4	3
144	0.1	3	2	5	2	4	2	3	2	2	5	3	6	2
5	1.9	3	2	3	3	3	3	3	3	3	3	3	1	3
67	0.1	2	3	1	3	2	5	5	3	3	3	2	3	5
35	0.3	1	3	2	3	2	4	4	4	2	3	3	2	3
40	0.2	1	3	2	4	1	4	4	4	4	3	4	5	3
1	10.1	1	3	2	5	3	5	4	4	4	3	3	5	4
22	0.5	1	3	3	3	2	3	2	3	3	1	3	3	4
4	2.6	2	3	3	3	2	3	2	3	3	1	3	3	4
14	1.1	3	3	4	3	3	3	3	3	5	3	5	3	3

Generalizable Interactive Streaming-REST Design

2017 UK Election 10 Day Design – Population Ideological Tree

Classification Tree



Id	%	Politicians JC	TM	AR	BJ	News/Bloggers In	Te	RP	P	KM	OJ	PM	LM	GF
9	1.4	5	5	6	6	5	6	5	6	6	5	6	6	6
7	1.6	'∞'	1	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'	'∞'
10	1.4	5	6	5	5	5	5	5	5	5	4	5	5	6
8	1.5	4	6	4	6	6	1	4	4	5	6	4	5	5
3	3.3	4	6	6	4	1	4	4	4	6	3	4	6	3
189	0.1	3	1	4	5	4	3	5	3	5	3	4	5	3
6	1.8	4	5	3	5	4	3	3	3	3	3	5	4	4
73	0.1	3	1	3	3	3	2	3	1	3	2	3	4	3
2	7.6	3	2	3	3	3	2	3	1	3	2	3	4	3
173	0.1	3	2	3	4	1	2	3	1	3	2	4	4	3
144	0.1	3	2	5	2	4	2	3	2	2	5	3	6	2
5	1.9	3	2	3	3	3	3	3	3	3	3	3	1	3
67	0.1	2	3	1	3	2	5	5	3	3	3	2	3	5
35	0.3	1	3	2	3	2	4	4	4	2	3	3	2	3
40	0.2	1	3	2	4	1	4	4	4	4	3	4	5	3
1	10.1	1	3	2	5	3	5	4	4	4	3	3	5	4
22	0.5	1	3	3	3	2	3	2	3	3	1	3	3	4
4	2.6	2	3	3	3	2	3	2	3	3	1	3	3	4
14	1.1	3	3	4	3	3	3	3	3	5	3	5	3	3

Generalizable Interactive Streaming-REST Design

2017 UK Election 10 Day Design – Top 25 $\texttt{sorted}_\downarrow$ Retweet Network Degrees

Screen Name	Out-degree	Out-nbhd	In-degree	In-nbhd
@jeremycorbyn	516833	184236	21	20
@OwenJones84	202084	78548	261	192
@Independent	195573	67341	681	22
@britainelects	130161	46921	15	14
@piersmorgan	118588	79514	157	128
@jonsnowC4	90555	53637	94	28
@paulmasonnews	74207	27358	309	222
@Telegraph	60732	29500	95	15
@LouiseMensch	53739	16916	3287	916
@Peston	48052	29552	25	8
@theresa_may	47791	31075	0	0
@faisalislam	46715	21148	101	75
@AngelaRayner	45272	15751	101	68
@DavidLammy	43043	27350	29	21
@davidallengreen	39141	15527	183	95
@bbclaurak	37683	18288	85	29
@IanDunt	36600	16069	203	157
@LordBuckethead	36436	28899	13	10
@Kevin_Maguire	36378	17015	5	1
@stephenfry	32521	26379	2	2
@Ed_Miliband	32264	23832	9	9
@MailOnline	31988	15781	594	10
@johnprescott	31906	23329	51	29
@GuidoFawkes	29033	10410	78	37
@MayorofLondon	27816	20162	44	17

All the chosen influencers, except Boris Johnson – the second most RT'd conservative MP (59) – are in top 25.

What's Happening in the Measurable Spaces Now?

Reporting couple examples from Apache Spark Summit 2017 in Dublin (that just finished yesterday):

- See data artist KeyNote talk yesterday by Jer Thorp (Twitter: @blprnt) on *Living in Data*
- Signals intelligence library – fully open sourced! by Antoine Amend and Andrew Morgan on *Story Deduplication and Mutation #EUstr9* (see their book: Mastering Spark for Data Science and fork at `https://github.com/PacktPublishing/Mastering-Spark-for-Data-Science`)

Some mathematical statistical challenges ahead

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Susceptible-Infected Contact Network (SICN) & Transmission Tree (TT)

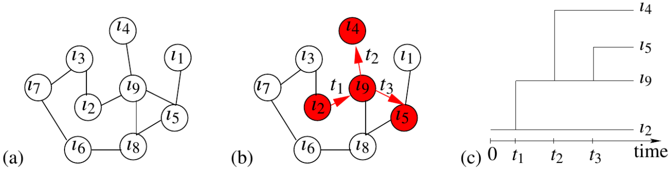

Figure 1: Spread of an epidemic over (a) the contact network of a population as shown by (b) a sub-network where edges representing transmission events are labelled by the time of event and the infected vertices are colored red and (c) the corresponding transmission tree.

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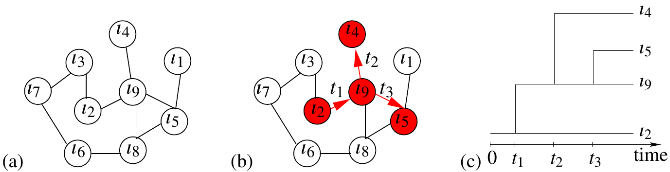

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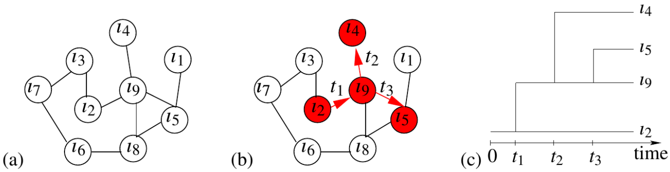

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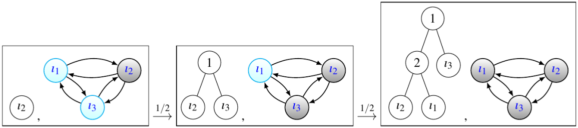
Aldous' ?n: How does the geometry or structure of the SICN afftect the distribution (shape and timing) of the TT?

Answer: It is possible in the simplest setting...

Markov chain on SI Contact Networks × Transmission Trees

- A growing transmission tree on a **complete** SICN in a population of size $n = 3$



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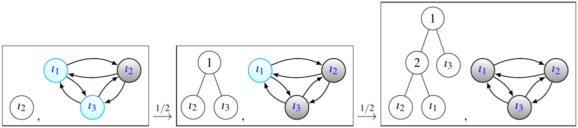

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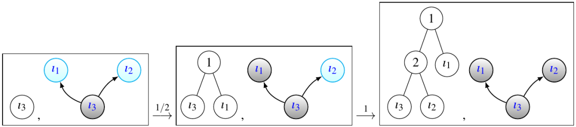

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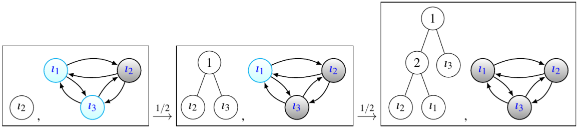

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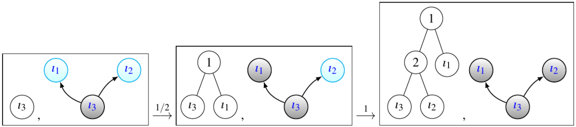
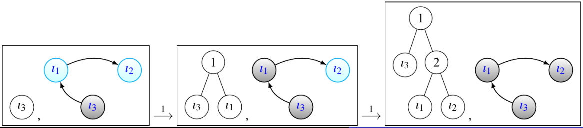


- A growing transmission tree on a **path** SICN in a population of size $n = 3$



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    - Note the poset on 2^{w_n} with unit weights given by $\prec := \subseteq$
    - So the current state of the Markov chain at discrete time z is $(\tau(z), c(z)) \in \mathcal{T}_n \times \mathcal{C}_n$

Transition Probabilities

- One-step transitions for the jump chain

$$\Pr\{(\tau(z+1), c(z+1)) \mid (\tau(z), c(z))\} =$$

$$\frac{\text{the edge-weight from } (z+1)\text{-th infector to the } (z+1)\text{-th infectee}}{\text{Sum of edge-weights from every potential infector to every potential infectee within its susceptible out-neighborhood}}$$

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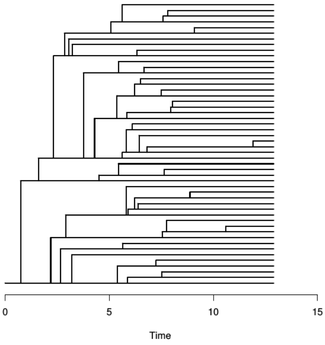
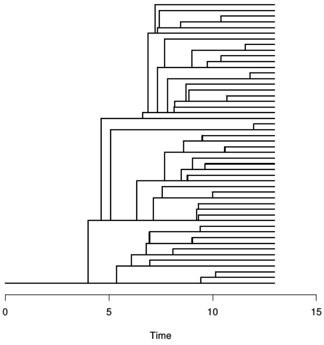
- NOTE: We limit to connected networks with unit weights and undirected edges here

Continuous Time Transmission Process

Two Transmission Trees (TTs) Grown on a Complete Susceptible-Infected Contact Network (SICN) with $n = 50$ individuals





Continuous Time Transmission Process

Branch-lengths of the TTs Grown on Complete SICNs randomly shifted logistic limit

for eg. (Aldous, 2013, Eq. 7.13)):

$$T_{\lfloor un \rfloor} - \log n \xrightarrow{d} F^{-1}(u) + G, \quad 0 < u < 1,$$

where, F is the logistic function:

$$F(t) = \frac{\exp(t)}{1+\exp(t)}, \quad -\infty < t < \infty$$

and G has Gumbel distribution with $\Pr(G < x) = \exp(e^{-x})$.

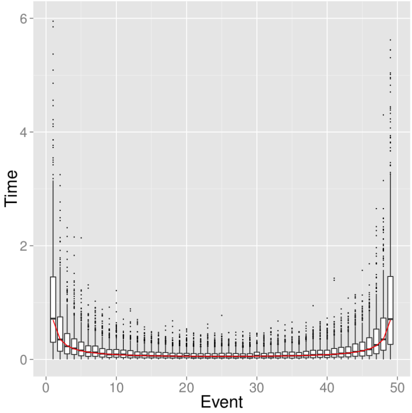


Fig. 6 The sampling distribution of T_z, branch-lengths (times in y-axis) of the transmission tree when there are exactly z infected individuals or between the $(z-1)$-th and z-th infection event (x-axis), where $z \in \{1, 2, \ldots, n-1\}$, from 500 independent simulations of the transmission tree over the complete SI contact network for a population of size $n = 50$ (as box plots) and the median branch-lengths given by $E(T_z)\log 2 = (\lambda z(n-z))^{-1} \log 2$, with $\lambda = 1/(n-1)$ (as red solid line).

Continuous Time Transmission Process

What is the distribution of transmission trees for an essentially arbitrary contact network?

Transmission Trees on Other Contact Networks

Mean branch-lengths for star, path and complete networks

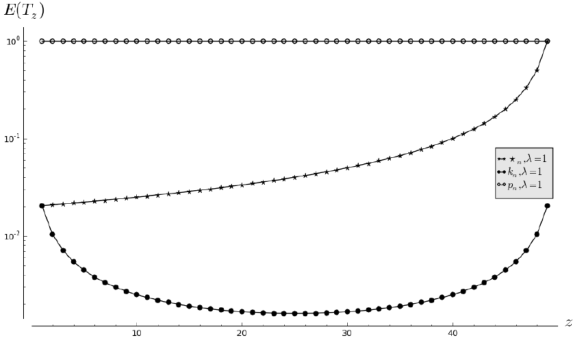
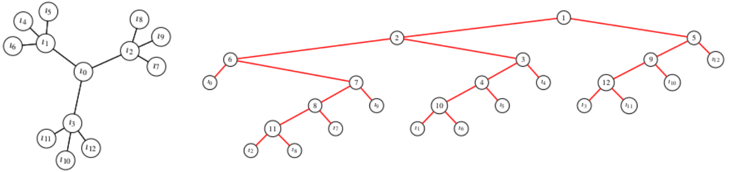
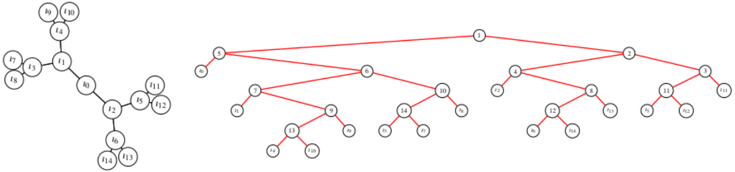


Figure 5: Expected branch-lengths when there are z infection events or $z+1$ infected individuals, $E(T_z)$, for the three cases. Here $n = 50$ and $\lambda = 1$. $E(T_z) = 1/\lambda = 1$ with the path network p_n of Sect. 2.1.3. $E(T_z) = 1/(\lambda(n-z)) = 1/(50-z)$ with the star network $\star_n$ of Sect. 2.1.2 and $E(T_z) = 1/(\lambda z(n-z)) = 1/(z(50-z))$ with the complete network k_n of Sect. 2.1.1 as z ranges in $\{1, 2, \ldots, n-1 = 49\}$.

Transmission Trees on Other Contact Networks

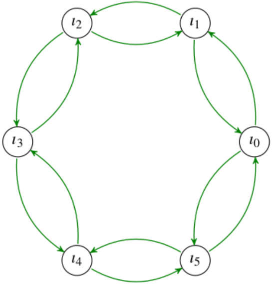
Trees on Balanced Tree Networks

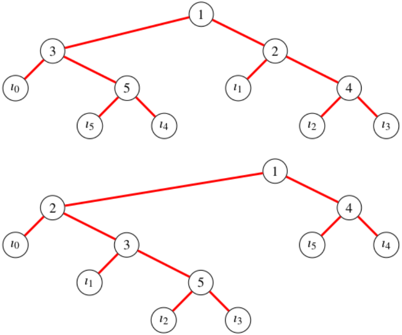




Transmission Trees on Other Contact Networks

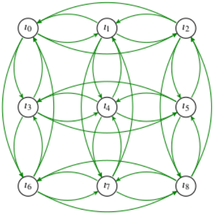
Trees on Bidirectional Circular Networks

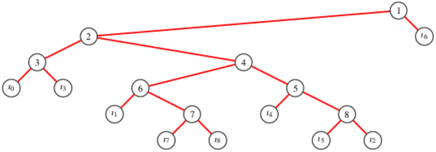


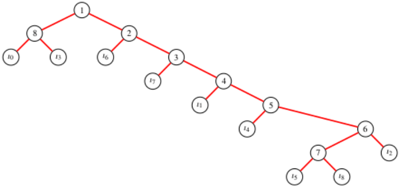


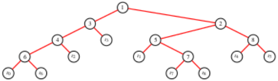
Transmission Trees on Other Contact Networks

Trees on Toroidal Network



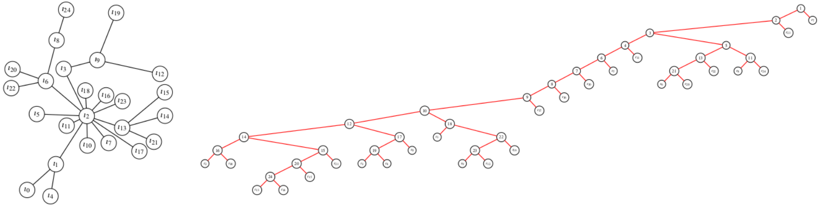






Transmission Trees on Other Contact Networks

Trees on Preferential Attachment Network



Transmission Trees on Other Contact Networks

Real-world networks are quite heterogeneous and are closer to mixtures of various families of deterministic and random contact networks...

What is the distribution of transmission trees for an essentially arbitrary contact network?

Beta-splitting Model

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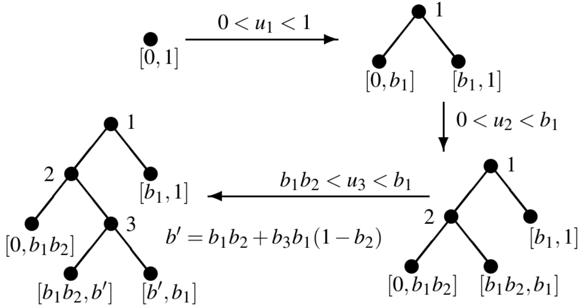
- Beta-splitting construction:



Theorems

Integrating out the interval-valued realizations at the leaf nodes

Theorem 1. *The probability of any discrete transmission tree* $\tau(m)$ *with* m *splits and* $m+1$ *leaves under the integrated Beta-splitting model is:*

$$\begin{aligned}
\Pr\{\tau(m)\} &= \prod_{z=1}^{m} \left\{ \frac{1}{B(\alpha+1,\beta+1)} \int_0^1 b_z^{s_z^L+\alpha}(1-b_z)^{s_z^R+\beta} db_z \right\} \times \Pr(\mathit{leaf\ labels}) \\
&= \prod_{z=1}^{m} \frac{B(s_z^L+\alpha+1, s_z^R+\beta+1)}{B(\alpha+1,\beta+1)} \times \Pr(\mathit{leaf\ labels}), \qquad (3.4) \\
&= \prod_{z=1}^{m} \left(\frac{\prod_{j=0}^{s_z^R} \frac{\beta+j}{\beta+j+\alpha} \prod_{i=0}^{s_z^L} \frac{\alpha+i}{\alpha+i+\beta+s_z^R+1}}{\frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}} \right) \times \Pr(\mathit{leaf\ labels}), \qquad (3.5)
\end{aligned}$$

Theorems

Beta-splitting model matches distrn on TTs for three example SICNs

- $(\alpha, \beta) = (0, 0) \equiv$ complete SICN,
- $(\alpha, \beta) \to (\infty, -1) \equiv$ star SICN
- $(\alpha, \beta) \to (-1, \infty) \equiv$ path SICN
- Theorem 2 on MLE expressions
- Theorem 3 on Equivalence class of initialized SICNs with the same (α, β)-specified TT distribution
- Will present a chalk talk in CIM on Feb 14 2017...
- 50 other model parameters simulated...

MLE of α and β from TTs under various SICNs

mean MLEs based on transmission trees simulated from various contact networks indexed by their ID from Table 1.

Beta-projections of various models

ID	Contact network	n	r	trials	$\bar{\hat{\alpha}}$ (s.e.)	$\bar{\hat{\beta}}$ (s.e.)
1	Complete	1,000	1	5	-0.006952 (0.06853)	0.05208 (0.1005)
2	Star	1,000	1	5	∞ (0.0000)	-1.0000 (0.0000)
3	Path	1,000	1	5	-1.0000 (0.0000)	∞ (0.0000)
4	Bidirectional Circular	50	1	5	-0.9880 (0.0006)	1.4584 (0.1534)
5	Bidirectional Circular	50	100	5	-0.9879 (0.0000)	1.5189 (0.0067)
6	BalancedTree(2,9)	1023	1	5	-0.4052 (0.0000)	-0.1477 (0.0000)
7	BalancedTree(3,6)	1093	1	5	-0.06452 (0.0000)	-0.5215 (0.0000)
8	BalancedTree(4,5)	1365	1	5	0.06556 (0.0000)	-0.7109 (0.0000)
9	BalancedTree(6,4)	1555	1	5	0.2350 (0.0000)	-0.8510 (0.0000)
10	BalancedTree(10,3)	1111	1	5	0.9249 (0.0000)	-0.9156 (0.0000)
11	BalancedTree(32,2)	1057	1	5	1.1624 (0.0000)	-0.9853 (0.0000)
12	BalancedTree(999,1)	1000	1	5	∞ (0.0000)	-1.0000 (0.0000)

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13	2D toroidal grid	1024	1	5	-0.8612 (0.008425)	-0.5606 (0.03219)
14	2D toroidal grid	10000	1	5	-0.89346 (0.0022)	-0.6626 (0.0106)
15	3D toroidal grid	1000	1	5	-0.6849 (0.01479)	-0.3515 (0.03451)
16	3D toroidal grid	10648	1	5	-0.7628 (0.007956)	-0.4968 (0.01641)
17	ER(100, 0.030)	100	30	5	-0.6063 (0.01383)	-0.4052 (0.02710)
18	ER(100, 0.040)	100	30	5	-0.5179 (0.01855)	-0.3151 (0.02244)
19	ER(100, 0.050)	100	30	5	-0.4059 (0.02020)	-0.2246 (0.01952)
20	ER(100, 0.10)	100	30	5	-0.1997 (0.03106)	-0.1280 (0.03063)
21	ER(100, 0.20)	100	30	5	-0.1074 (0.03961)	-0.06166 (0.03020)
22	ER(100, 0.40)	100	30	5	0.02247 (0.06603)	0.01541 (0.05499)
23	ER(100, 0.64)	100	30	5	-0.01097 (0.03984)	0.01046 (0.05112)
24	ER(100, 1.0)	100	30	5	-0.001787 (0.04347)	-0.01555 (0.04019)

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25	RandReg(1000, 3)	1000	1	5	-0.7504 (0.004186)	-0.06260 (0.06322)
26	RandReg(1000, 4)	1000	1	5	-0.5530 (0.04513)	-0.002305 (0.09785)
27	RandReg(1000, 6)	1000	1	5	-0.3520 (0.03464)	0.06042 (0.06586)
28	RandReg(1000, 10)	1000	1	5	-0.1939 (0.06167)	0.07274 (0.1238)
29	RandReg(1000, 100)	1000	1	5	0.06378 (0.04519)	0.1084 (0.05844)
30	RandReg(1000, 999)	1000	1	5	-0.01496 (0.08893)	0.006464 (0.04166)
31	$\text{SWRN}^{*,\circ}(50, 2, 0.0)$	50	30	5	-0.9878 (0.0001516)	1.514 (0.01222)
32	$\text{SWRN}^{*}(50, 2, 0.1)$	50	30	5	-0.9618 (0.003047)	-0.4147 (0.03203)
33	$\text{SWRN}^{\circ}(50, 2, 0.1)$	50	30	5	-0.9652 (0.002863)	-0.3828 (0.1171)
34	$\text{SWRN}^{*}(50, 2, 0.2)$	50	30	5	-0.9375 (0.004620)	-0.5683 (0.0193)
35	$\text{SWRN}^{*}(50, 2, 0.5)$	50	30	5	-0.8632 (0.008181)	-0.6471 (0.03722)
36	$\text{SWRN}^{*}(50, 5, 0.1)$	50	30	5	-0.7530 (0.01572)	-0.4751 (0.04671)

MLE of α and β from TTs under various SICNs

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Beta-projections of various models

36	SWRN*(50,5,0.1)	50	30	5	-0.7530 (0.01572)	-0.4751 (0.04671)
37	SWRN$^\circ$(50,5,0.1)	50	30	5	-0.7918 (0.01596)	-0.5130 (0.03323)
38	SWRN$^\circ$(50,5,0.2)	50	30	5	-0.6881 (0.03277)	-0.3595 (0.06002)
39	SWRN$^\circ$(50,5,0.5)	50	30	5	-0.5264 (0.04687)	-0.2138 (0.09471)
40	SWRN$^\circ$(100,2,0.2)	100	1	5	-0.9479 (0.01509)	-0.3991 (0.5065)
41	SWRN$^\circ$(100,2,0.2)	100	30	5	-0.9493 (0.003869)	-0.6027 (0.03475)
42	SWRN*(100,2,0.5)	100	1	5	-0.9023 (0.03411)	-0.7139 (0.03929)
43	SWRN*(100,2,0.5)	100	30	5	-0.8878 (0.006687)	-0.6821 (0.02189)
44	SWRN$^\circ$(100,2,0.5)	100	1	5	-0.8714 (0.05584)	-0.6533 (0.09257)
45	SWRN$^\circ$(100,2,0.5)	100	30	5	-0.8920 (0.005128)	-0.6786 (0.02189)
46	SWRN$^\circ$(100,5,0.99)	100	30	5	-0.5079 (0.02371)	-0.2290 (0.03059)
47	SWRN$^\circ$(100,10,0.99)	100	30	5	-0.2027 (0.07641)	-0.05611 (0.06949)

MLE of α and β from TTs under various SICNs

mean MLEs based on transmission trees simulated from various contact networks indexed by their ID from Table 1.

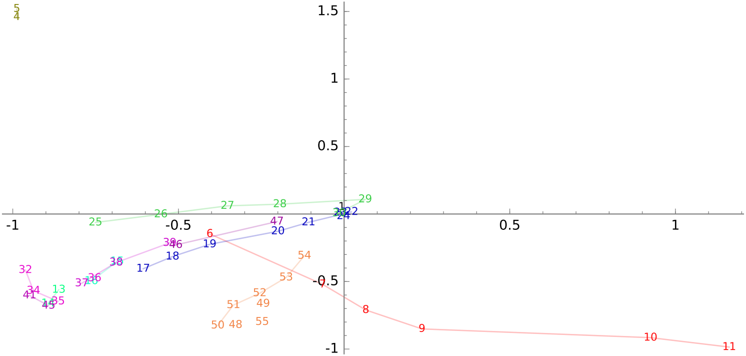
Beta-projections of various models

48	PrefAttach$^*(100,1)$	100	30	10	-0.3275 (0.04932)	-0.8215 (0.01121)
49	PrefAttach$^*(100,2)$	100	30	10	-0.2443 (0.03283)	-0.6647 (0.01294)
50	PrefAttach$^\circ(100,1)$	100	30	10	-0.3813 (0.04908)	-0.8254 (0.005460)
51	PrefAttach$^\circ(100,2)$	100	30	10	-0.3339 (0.03884)	-0.6743 (0.01657)
52	PrefAttach$^\circ(100,3)$	100	30	10	-0.2545 (0.04181)	-0.5863 (0.01652)
53	PrefAttach$^\circ(100,5)$	100	30	10	-0.1748 (0.04214)	-0.4698 (0.03110)
54	PrefAttach$^\circ(100,10)$	100	30	10	-0.1196 (0.03449)	-0.3089 (0.02663)
55	PrefAttach$^\circ(100,1)$	100	1	5	-0.2472 (0.2698)	-0.7993 (0.05843)

MLE of α and β from TTs under various SICNs

mean MLEs based on transmission trees simulated from various contact networks indexed by their ID from Table 1.

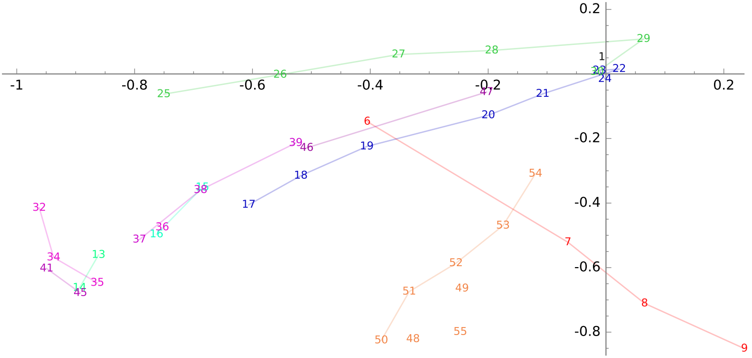
Beta-projections of various models



MLE of α and β from TTs under various SICNs

mean MLEs based on transmission trees simulated from various contact networks indexed by their ID from Table 1.

Beta projections of various models



So what? — We can do Bayesian non-parametrics by Beta-splitting mixtures over empirical SICNs/TINs

The End

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  3. Centre for Mathematics and its Applications, Ecole Polytechnique, Palaiseau, France.
- Code Contributors: Ivan Sadikov, Akinwande Atanda and Joakim Johansson

- The Transmission Process: A Combinatorial Stochastic Process for the Evolution of Transmission Trees over Networks, Raazesh Sainudiin and David Welch, Journal of Theoretical Biology, Volume 410, Pages 137–170, 2016 10.1016/j.jtbi.2016.07.038
- Seeded by Hate? Characterizing the Twitter Networks of Prominent Politicians and Hate Groups in the 2016 US Election, Kumar Yogeeswaran, Kyle Nash, Rania Sahioun and Raazesh Sainudiin, 2017 http://lamastex.org/preprints/2017HateIn2016USAElection.pdf

- See: Project MEP for more information: http://lamastex.org/lmse/mep