Hawkes Processes on Social and Mass Media: A Causal Study of the #BlackLivesMatter Movement in the Summer of 2020

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Abstract: In this work we study interactions in social media and the reports in mass media during the Black Lives Matter (BLM) protests following the death of George Floyd. We implement open-source pipelines to process the data at scale and employ the self-exciting counting process known as Hawkes process to address our main question: is there a causal relation between interactions in social media and reports of street protests in mass media? Specifically, we use distributed label propagation to identify such interactions in Twitter, that supported the BLM movement, and compared the timing of these interaction to those of news reports of street protests mentioning George Floyd, via the Global Database of Events, Language, and Tone (GDELT) Project. The comparison was made through a Bivariate Hawkes process model for a formal hypothesis test of Granger-causality. We show that interactions in social media that supported the BLM movement, at the beginning of nationwide protests, caused the global mass media reports of street protests in solidarity with the movement. This suggests that BLM activists have harnessed social media to mobilise street protests across the planet.

1 INTRODUCTION

On 25th of May 2020, George Floyd, a 46 year old African-American man, is arrested in Minneapolis, Minnesota for allegedly using a counterfeit $20 bill to buy cigarettes. The arrest is caught on film by passersby, showing how police officer Derek Chauvin pins the handcuffed Floyd to the ground with his knee on Floyd's neck, while his three colleagues prevent anyone from intervening. Floyd repeatedly utters the words "I can't breathe" before he goes unconscious. He later dies at the hospital, and the video of the arrest goes viral on Facebook (Deliso, 2021). The next day protests in support of the Black Lives Matter (BLM) movement, and against police brutality, start in Minneapolis, which during the following days will spread both nationally and internationally to over 60 countries, and become what may be the largest protests in U.S. history to date, with polls estimating attendances in the range of 15-26 million people (Buchanan et al., 2020).

BLM is a decentralised grassroots movement that began on social media, using the hashtag #BlackLivesMatter in the wake of the shooting of Trayvon Martin in July 2013. The movement has since then gained attention for demonstrations following the deaths of Michael Brown and Eric Garner in 2014, and George Floyd in 2020, with its main issues being that of advocating against police brutality toward African-Americans, and policy issues related to racial injustices (Jackson et al., 2020).

As reactions and critiques of the BLM movement, the phrase "All lives matter" was coined, as well as the phrase "Blue lives matter", after the shooting of two police officers during protests in Ferguson, Missouri in 2015. Both of these slogans are associated with conservative views, and rejects the BLM-movement's idea of a need to focus on the racial injustice towards African Americans.

The decentralised nature of all three of these movements, and the way social media has played a key part in their development, leading to real life events such as mass protests, motivates our choice to analyse data from social media and from mass media to try to get a better understanding of the mobilisation in social media into real-world action.

In this work we study the landscape in mass and social media during the first month of protests that followed after the murder of George Floyd. Our primary question is whether there is a statistically significant

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interaction between communications in socially networked communities and street protests as measured by published reports in mass media. We attempt to answer this question by devising a data processing framework to mathematically model the interactions between social and mass media via the family of point processes known as Hawkes processes and conduct statistical hypothesis tests of Granger causality, subsequent to identifying influential social media communities using network models.

The paper's outline is as follows. We describe Models in Section 2, Data Handling in Section 3, Analysis of Twitter Data in Section 4, Joint Media Modeling in Section 5 and conclude in Section 6.

2 MODELS

2.1 Hawkes Processes

We will now introduce a family of point processes known as Hawkes processes, assuming the reader is familiar with point processes. These processes were introduced by Hawkes (Hawkes, 1971), and due to their self-exciting nature they are used in fields such as epidemiology, seismology, and finance (Daley and Vere-Jones, 2003; Bacry et al., 2015).

Suppose we observe events in continuous time, i.e., points on the positive real line as *timestamps*, where for each i, t_i is the exact time where some sort of event occurs for the i-th time. Define the *history* of a point process up to time t, as the set $\mathcal{H}_t$ containing all timestamps $\{t_i\}$ up to time t. A Hawkes process allows us to model the occurrence of future events after time t based on the entire history $\mathcal{H}_t$ up to time t as follows:

Definition 2.1. Let $N(t)$ be a point process that counts the number of events up to time t with history $\mathcal{H}_t$. If the intensity $\lambda(t)$ of $N(t)$ is of the form

$$\lambda(t) = \mu + \sum_{t_i \in \mathcal{H}_t} \phi(t - t_i) \ , \tag{1}$$

we define $N(t)$ as a *Hawkes process*, where μ is the *baseline intensity* and $\phi(t)$ is the *kernel*.

We will now introduce a particular choice of kernel.

Definition 2.2. We define

$$\phi(t) = \alpha\beta e^{-\beta t} \ , \tag{2}$$

as an *exponential kernel* where parameter $\alpha \geq 0$ is the *self-excitation parameter*, and parameter $\beta > 0$ is the *decay rate*.

Parameter α thus decides how much an occurred event will influence the rate of new events, while β will decide how long into the future this influence will last as $\phi(t) \to 0$, when $t \to \infty$.

A natural extension of the Hawkes process is the multivariate Hawkes process.

Definition 2.3. Let $d \in \mathbb{N}$ be the number of dimensions, and $\mathcal{H}_{t,i}$ for $i = 1,..,d$ be the history of events in dimension i. The multivariate point-process induced by the intensities

$$\lambda_i(t) = \mu_i + \sum_{j=1}^{d} \sum_{t_k \in \mathcal{H}_{t,j}} \phi_{ij}(t - t_k) \quad i = 1,...,d \tag{3}$$

is then defined as a *multivariate Hawkes process*. If the kernel $\phi_{ij}(t)$ takes the form of the following *multivariate exponential kernel*,

$$\phi_{ij}(t) = \alpha_{ij}\beta_{ij}e^{-\beta_{ij}t} \quad i,j = 1,...,d \ , \tag{4}$$

where $\alpha_{ij} \geq 0$ is the *excitation parameter*, and $\beta_{ij} > 0$ is the *decay rate*, then we have the *multivariate Hawkes process with exponential kernel*.

The excitation parameter α_{ij} can be interpreted similarly as α in the one-dimensional case with the exponential kernel, with the exception that this influence on new events in dimension i now may come from previous events in any dimension $j \in \{1,\ldots,d\}$. Analogously, β_{ij} is interpreted as the rate of decay that specifies how past events in dimension j can influence the arrival of new events in dimension i. In Section 5 we use a multivariate Hawkes process to model Twitter events in dimension 1 and mass media reports of protests in dimension 2.

2.2 Granger Causality

How to rigorously define causality has been a topic of discussion in western philosophy for over 2000 years, starting with Plato and Aristotle (Falcon, 2019), and continuing on with Hume and Kant's disagreement being one of the fundamental discussions in modern philosophy. The problem is still open, (Pierris and Friedman, 2018).

In light of this, and in some sense to get around the metaphysical complications of proper causality, Clive Granger introduced the concept of *Granger Causality* relating to stochastic processes. The basic idea is if a variable X_t *Granger-causes* variable Y_t, then the past values of X_t contain information that helps predict future values of Y_{t+1} better than doing prediction based only on past values of Y_t (Granger, 1980).

Using the following Theorem from Eichler (Eichler et al., 2012), we will test the null hypothesis of the non-existence of Granger causality between events in social and mass media, and vice versa, in the sequel.

Theorem 2.1. *Let $N(t)$ be a multivariate Hawkes process in d dimensions, with kernels $\phi_{ij}(t)$, $i, j \in \{1, \ldots, d\}$. Then the j-th component N_j does not Granger-cause the i-th component N_i if and only if $\phi_{ij} = 0$, $\forall t \in \mathbb{R}$.*

Thus, when $N(t)$ is a multivariate Hawkes process with exponential kernel, by Theorem 2.1 the j-th component N_j does not Granger-cause the i-th component N_i if and only if $\alpha_{ij} = 0$, $\forall t \in \mathbb{R}$.

3 DATA HANDLING

3.1 Apache SPARK

The data was handled using Apache Spark[1] which is an open-source engine designed for data engineering, data science, and machine learning on clusters of multiple computers, by implicit data parallelism. Spark is multi-language and supports `Scala`, `Python`, `R`, `SQL`, `Java`, `C#` and `F#`. While most of the code for this article was written in `Scala`, the ease of switching between languages in the same environment proved quite useful, as we would use libraries written in both `R` and `Python` .

On top of Spark core, Spark SQL (Armbrust et al., 2015), which introduces the data abstraction of DataFrames, allows manipulation in `Scala`, `Python`, and `R` using the standard `SQL` language, and the graph-processing framework GraphX (Gonzalez et al., 2014), allows for network-analysis. To run Spark, the cloud data platform Databricks was used, which provided cloud storage, computing clusters, and a notebook-environment to write and run the code after loading the two main libraries developed for this study, `MEP`[2] and `SPARK-GDELT`[3].

3.2 Twitter

Twitter is a micro-blog and social media service, founded in 2006, where users post and interact via *tweets* – a short message restricted to 280 characters, which may also contain pictures, short videos and URLs. Tweets can be original posts, replies to other tweets, or retweets, i.e., sharing of another user's tweet. As long as a user does not actively chose to be private, anyone is able to read the tweets of the user. To help a tweet gain attraction, and make it easier for other users to find tweets on a specific topic, the user can tag their posts by including keywords prefaced with '#', the hash symbol. These tagged keywords are called *hashtags* and they have been used by activists in global social movements such as #BlackLivesMatter and #MeToo (Jackson et al., 2020).

Users may also follow other users on Twitter. The relationship of following is asymmetrical, meaning that if user A follows user B, user B does not have to follow user A. Compare this to Facebook, where users mutually have to accept each other as friends to be able to interact. To simplify things, if Facebook is about keeping in touch and networking with your friends, Twitter is about sharing and receiving information the user finds interesting; according to a study done in 2014, 44% of Twitter's users have never tweeted which seems to suggest that a large part of the user base only uses Twitter for receiving information (Muphy, 2014). Due to this asymmetrical following relationship, which encourages a more open discourse between users, along with its magnitude of users, choosing Twitter as the social media to analyse becomes the natural choice. Furthermore, unlike Twitter, other prominent social media platforms including Facebook and Instagram do not allow researchers open access to their data. We developed `MEP` to be able to design experiments, collect and analyse data from different Twitter APIs at scale in public cloud infrastructure.

3.2.1 Application Programming Interface

To work with and be able to analyse Twitter data efficiently on an arbitrarily large scale, access to Twitter's Application Programming Interface (API) is needed, and requires Twitter developer credentials, which anyone can apply for. With access to the credentials, one may request and download tweets which can be represented as `JSON`-files. At the time of writing, two versions of the Twitter API exists. This work was done in the older version 1.

To get a sense of how the data was handled, a brief overview of the relevant fields from the schema of the `JSON` for a tweet will be presented. For full details, we refer to Twitter's data dictionary[4] [5]. The two most basic objects for a tweet are the User object and the Tweet object shown in Tables 1 and 2, respectively.

From the User object, as the name suggests, we get access to the metadata of a user. However, note that no direct information about which users follow the user, or which users the user follows, beyond the counts, is accessible from the user object.

[1] https://github.com/apache/spark

[2] https://github.com/lamastex/mep

[3] https://github.com/lamastex/spark-gdelt

[4] https://developer.twitter.com/en/docs/twitter-api/v1/data-dictionary/object-model/tweet

[5] https://developer.twitter.com/en/docs/twitter-api/v1/data-dictionary/object-model/user

Table 1: Some attributes, with their types and description, for the User object.

User object		
Attribute	Type	Description
id	Int64	The unique integer representation of the user.
screen _name	String	The screen name, also known as handle of the user.
followers _count	Int	The number of followers the user has.
friends _count	Int	The number of users the user follows.

From the Tweet object, we get access to the metadata of a tweet. Via the field "user", we also get the information of the user behind the tweet, since this is a User object. Moreover, since the fields "quoted_status" and "retweeted_status" are Tweet objects, we get the full information of the original post that has been retweeted or quoted.

Note that the Tweet Object in "retweeted_status" points to the original tweet that has been retweeted, if the post is a retweet. It is possible for a user to retweet another user's retweet, but information on this chain of events is thus not accessible. For example, let user A write a tweet T that gets retweeted by user B. Later, user C sees this retweet on user B's timeline and then retweets T. Twitter's API will then only tell us that user B and C have retweeted user A, but not the fact that user C accessed this tweet via user B. This limitation also motivates the use of retweet network in Section 4.2.

Along with these two objects, there is another object named entities, which contains all the metadata of a tweet's content, including any URLs, hashtags, twitter handles of users mentioned, and media content (pictures and short video clips).

3.2.2 Data Set

The data set that was used (Giorgi et al., 2020) has 41.8 million collected tweets from 10.1 million unique users regarding the Black Lives Matter movement, along with the smaller counter movements of Blue Lives Matter (pro-police movement) and All Lives Matter. These tweets were collected by filtering on the keywords: *BlackLivesMatter*, *BlueLivesMatter* and *AllLivesMatter*. The data contains tweets from the beginning of the movement in 2013 to 30 June 2020. In this work, we focus on the events occurring during the aftermath of the death of George Floyd on 25 May 2020, and discard all tweets before this date.

3.2.3 Collecting Data

Due to Twitter's policy, collecting and sharing tweets publicly is not allowed. To share a set of tweets, instead one shares the IDs of each tweet, and to get the full metadata of the tweets, access to Twitter's API is needed. There is also a limit on how many tweets one may collect per hour, which initially was a problem. To get around this, the `python` library `twarc`[6] was used. `twarc` allowed us to collect tweets from the IDs (a process known as *hydrating*), in an optimised way with respect to the hourly collection limit.

To be able to work with the data in Databricks and Spark, a Docker-container with `python` and `twarc` was set up on a remote machine, that ran the hydration script on small batches of the IDs, collected them as '.json'-files, and then compressed and stored them in our Databricks cloud storage. This procedure took roughly five days.

A consequence of retroactively collecting tweets from their IDs is that all tweets that have been removed due to various reasons (such as the users of these tweets getting banned, removing their accounts, or going private) at the time of hydrating, are not accessible and were therefore not collected.

After hydrating the IDs from the data set, and discarding tweets posted earlier than 24 May 2020, 23.3 million tweets from 7.1 million unique users were left. These were cleaned to be easier to work with using Spark's Dataframes. We also categorised each tweet as an original tweet, retweet, quoted tweet, etc., and then stored them in the column-based data-storage format parquet on a delta lake (Armbrust et al., 2020). See `MEP` for details of the collector, preprocessor and categoriser behind the delta lake.

3.3 GDELT

The *Global Database of Events, Language, and Tone* (GDELT) project, founded in 2013, is an open database supported by Google Jigsaw, that monitors news media in print, broadcast, and web formats from all over the world in over 100 languages. It is updated every fifteen minutes and stretches back to the 1st January of 1979, containing meta-data such as the people and organisations being mentioned, events and their locations, counts of key-words along with the tone and emotions of the parsed news sources[7]. We used the GDELT database to get a high level understanding of the mass media landscape during the given time span, by reducing the records of reported events of protests, to data points in time. We accomplish this

[6] https://twarc-project.readthedocs.io/en/latest/

[7] https://www.gdeltproject.org/

Table 2: Some attributes, with their types and description, for the Tweet object.

Tweet object		
Attribute	Type	Description
created_at	String	UTC-time when the tweet was created.
id	Int64	The unique integer representation of the tweet.
text	String	The textual content of the tweet.
in_reply_to_status_id	Int64	If the tweet is a reply to another tweet, the field will contain the tweet-ID of that tweet. Otherwise null.
in_reply_to_user_id	Int64	If the tweet is a reply to another tweet, the field will contain the user-ID of that tweet. Otherwise null.
user	User Object	All information of the user of the tweet.
quoted_status	Tweet Object	If the tweet is a quote tweet, all information of the original tweet will be contained in this field. Otherwise null
retweeted_status	Tweet Object	If the tweet is a retweet, all information of the original tweet will be contained in this field. Otherwise null

by building an analytics-ready Delta Lake (Armbrust et al., 2020). A brief overview of GDELT to appreciate how we handled the data for this work follows. For a more thorough overview, we refer to the documentation[8] and `SPARK-GDELT`, our open-source library developed for this study.

3.3.1 Coding

The idea behind GDELT is that of *coding*, which is fundamentally fairly simple. Given a *record* – for example a written news article – go through the text and identify the real world *events* that are being reported in the record, and identify the *actors* who are involved in the event. During the Cold war, two coding frameworks dominated: WEIS and the *Conflict and Peace Data Bank*, COPDAB. Both of these frameworks, being developed and used in a 20th century post-World War II context, were focused on codifying how sovereign states (the actors) interacted through official diplomacy and military threats (Schrodt, 2012). For example, in the following sentence:

*"**President Reagan** has **threatened** further action against the **Soviet Union** in an international television program beamed by satellite to more than 50 countries"*,

one would identify the act of threatening as the event, and assign it some integer (decided by the code framework), with the actors being President Reagan (or the United States if the coder is only interested in sovereign states), and the Soviet Union.

This process of coding would historically be done by hand. However, the combination of psychological studies showing that the kind of sustained decision-making involved in coding leads to fatigue, inattention, and heuristic shortcuts, and the technological advancement in computing software and hardware, coding is nowadays automated. The frameworks for codifying has also developed since the cold war, with GDELT using the framework of *Conflict and Mediation Event Observations* (CAMEO) (Leetaru and Schrodt, 2013). Some notable changes being that actors are no longer limited to sovereign states, and include persons, organisations, and companies.

In practice, GDELT is essentially two separate but interlinked databases: The *Global Knowledge Graph* (GKG), which consists of records and the *Event Database*, which as the name suggests stores events that are being reported.

3.3.2 GKG

The Global Knowledge Graph (GKG) consists of all records from multiple news sources in the world. As of version 2 of GDELT, new records get added every fifteen minutes. Whenever a record is added, the source text is parsed via natural language processing to identify the events (using coding), locations, persons and organisations, as well as themes mentioned in the text. Moreover, keywords such as "protest" that are mentioned multiple times gets counted. Sentiment analysis is also incorporated to get a value of the tone of the source text (whether the text is positive, neutral or negative). Many other metadata extracts are in each GKG record.

3.3.3 Event Database

The Event database attempts to record all unique events that are being identified in the parsing process of the GKG database. Each data point is given a unique ID for the event, and contains the date, the actors along with the code of the type of event be-

[8] http://data.gdeltproject.org/documentation/GDELT-Global_Knowledge_Graph_Codebook-V2.1.pdf

ing identified. The coded event also gets mapped to the Goldstein-scale (Goldstein, 1992), which seeks to measure the potential impact the event could have on the stability of the country. Moreover, the Event database has metadata on how often the event has been mentioned by records in GKG and the average tone of these records.

3.3.4 Handling of the GDELT Data

Due to the sheer magnitude of data contained in the GDELT database, working with data proved quite a challenge. Our goal was to filter out the events about the protests relating to the Black Lives Matter movement and the counter movements between 25 May 2020 and 30 June 2020. Although the parsing of news records into the GKG database identifies organisations, it did not identify the Black Lives Matter movement as one, probably due to its lack of centralisation.

What we did instead was to filter out all data relating to protests happening in the world. This naturally led to noisy data, since we got reports of protest unrelated to the BLM movement, but we justify this by the fact that no other major protests were happening in the world at the same time. To check this, we filtered the Event database by events with CAMEO root-code 14, i.e., those events coded as protests, over a three months timeline.

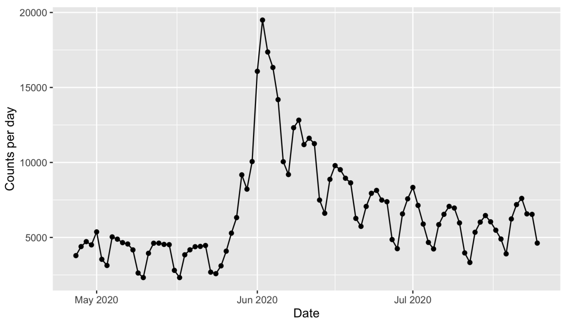

Figure 1: Events coded as protests in the GDELT Event database.

As we see in Figure 1, there is a baseline of roughly 5,000 events per day coded as protests before 25 May. This number then explodes, and there is nothing that suggests that the sudden increase in magnitude of protests are not related to the BLM protests. It is worth pointing out that there is no bijection between the real world protest and the protest data from the Event database. For example, if in one city during one day, large protests are taking place and one group of people are protesting peacefully while another group is rioting, then the coding framework should identify the act of the peaceful and rioting protesters as two different events (Schrodt, 2012), although they are near each other in time and space. Thus, saying that more than 8,000 protests happened on the 1 June 2020, would be incorrect.

In Section 5 we will look at news reports in mass media, and therefore use data from the GKG database. We did this by filtering by the themes of the records. All records in the GKG database with theme "PROTEST" were filtered out.

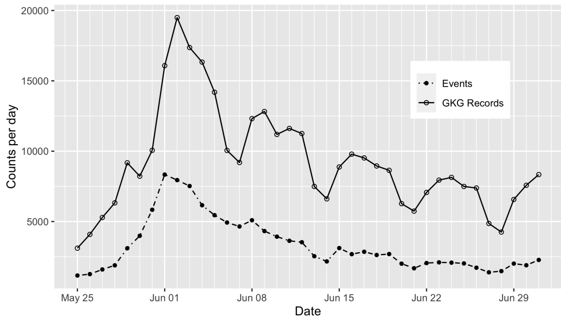

Figure 2: Comparison of records from the GKG database with theme "PROTEST", and events coded as protests from the Event database.

Ignoring the periodic dips in the GKG plot in Figure 2 (which are due to less reporting being done on weekends), the two plots follow a similar pattern. Naturally, there are more records than events, since multiple news sources may report the same event.

4 ANALYSIS OF TWITTER DATA

In this Section, we explore the Twitter data, first via simple querying on the data set, and then by doing network analysis on the induced retweet network. The results from this exploratory data analysis then motivated the choice of using Hawkes processes to model and perform hypothesis tests to shed light on the phenomena of interest in this study – occurrence of tweets in support of the BLM movement and that of mass media reports of street protests.

4.1 Data Observations

4.1.1 Timeline

We started by examining the data over the relevant time-span from 24 May 2020 to 30 June 2020. During this period, 23,346,745 tweets by 7,111,140 unique users were collected using `twarc` on the BLM data set (Giorgi et al., 2020).

From Figures 3 and 4, we can see that activity first starts on Twitter, and the reports of protests start to drastically increase on 27 May. We also see a dip in

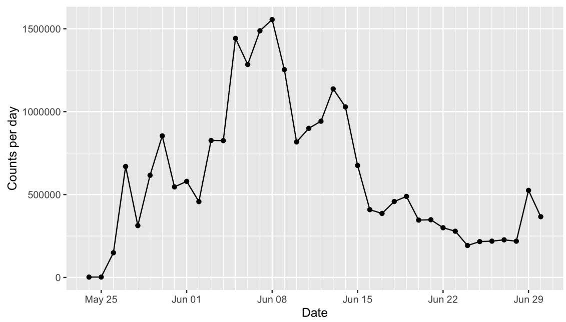

Figure 3: Number of tweets per day.

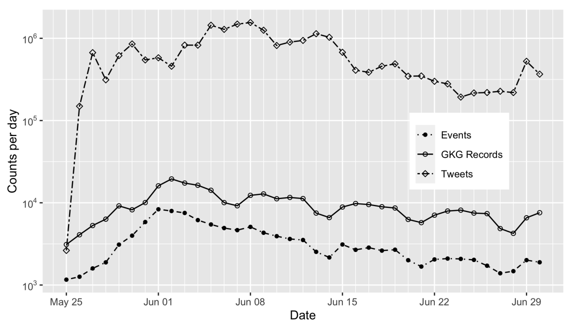

Figure 4: Log-scaled plot of the number of tweets, records and events

Twitter activity between 31 May and 2 June, while the GDELT data on the number of reports of protests spikes during these days. The explanation of this is simply that the data set lacks tweets on these days. This was found while exploring the data, and noticing that the data set contained retweets of a tweet from this time period, but not the original tweet. Whether these missing tweets disappeared during the collecting of data, or if they are missing in the original data set (Giorgi et al., 2020) of the Tweet IDs, remains unclear. To deal with this, we refrained from doing any modelling with tweets from this time period.

4.1.2 Type & Media Content of Tweets

Next, we examined TweetTypes, i.e., the types of status update or interactions in our Twitter data. The most to least frequent TweetTypes (% of data) were Retweets (55%), Retweets of Quoted Tweets (27%), Original Tweets (7%), Quoted Tweets (7%), Reply Tweets (3%), Original Tweets (1%). Thus, only 18% of the tweets in the BLM-data set were original tweets (either original, or replies to other tweets), with the remaining 82% being some sort of retweeted content. This suggests that the re-sharing of other users' original content is fundamental for how users interact with each other on Twitter, and motivated our choice of examining the retweet network.

One initial idea was to focus on URLs to news articles shared by Twitter users, and then link them to the GDELT database. However, we soon discovered that users in general did not share news sources from mass media. Instead highly retweeted tweets often contained original media (i.e., videos and pictures), which were often taken from the protests. For instance, 53% of tweets with over 1000 retweets, as opposed to only 17% of all tweets, shared original media.

4.2 Network Analysis

Section 4.1.2 showed the importance of retweets in the Twitterverse. In this Section we will formalise this by introducing a retweet network structure on our data set.

4.2.1 Retweet Network

Definition 4.1. Let $G_I = (V, E)$ be a directed weighted graph in time interval $I \subset \mathbb{R}_+$, where every vertex $v \in V$ is a unique Twitter user, and every edge $e \subset \{(u,v) \mid (u,v) \in E \subset V^2\}$ is interpreted as user v having retweeted u during time interval I. The weight $W(e) = W((u,v)) \in \mathbb{N}$ is the number of times user v has retweeted user u. We then define G_I as a *retweet network*.

Furthermore, we define G_I' as an *undirected retweet network* if $(u,v) \in E \Leftrightarrow (v,u) \in E$. Thus G_I' ignores whether u retweeted v or vice versa but preserves the information that there is a retweet relation between the two users.

We chose to look at retweets since a retweet by user u of an original tweet by user v is highly likely to mean that user u agrees with user v. Direct retweets are generally recognized to indicate trust in the communicator and endorsement (Jansen et al., 2009; Metaxas et al., 2015; Boyd et al., 2010). The number of times a user has been retweeted also gives a probabilistic interpretation, using the random geometric graph interpretation in (Sainudiin et al., 2019), that measures how influential a user is on another in terms of the lengths of their most retweeted paths.

By looking at our retweet network we can already get some information from the Twitter data set; simply by summing the outgoing edges and their weights for every user, we get the most retweeted users in our time interval between 24 May 2020 and 31 June 2020.

One noteworthy user is the sixth most retweeted user @MrAndyNgo. Andy Ngo is an American conservative journalist and a prominent opponent of the Black Lives Matter movement, who in February 2021 published *Unmasked: Inside Antifa's Radical Plan to*

Table 3: Ten most retweeted users, sorted by number of retweets. Usernames for non-public users have been anonymized. The communities were identifed using the label propagation algorithm.

Username	| followers |	| retweets |	Community
@JoshuaPotash	142,833	759,572	Pro-BLM
@YourAnonCentral	5,862,927	529,431	Pro-BLM
-	1,584	187,065	Pro-BLM
@elijahdaniel	760,935	161,337	Pro-BLM
-	22,983	135,698	Pro-BLM
@MrAndyNgo	799,291	125,898	Anti-BLM
-	1,232	125,826	Pro-BLM
@BTS_twt	34,107,446	125,534	K-pop
@shawnwasabi	140,788	106,731	Pro-BLM
@Drebae_	141,613	103,594	Pro-BLM

Destroy Democracy (Ngo, 2021), where he among other things writes about his experiences from the BLM protests of 2020. His presence amongst the most retweeted users will serve as a gateway into the counter-movements of All Lives Matter and Blue Lives Matter. Thus, we need to detect different communities within the observed retweet network, such that each community has more edges or retweets within it when compared to the number of edges between it and another community.

4.2.2 Connected Components

The motivation behind the definition of an undirected retweet network follows in the next step, when we look at the connected components of our graph.

Definition 4.2. Let G be a graph. A sequence of edges $(e_1,...,e_{n-1})$ is called *path* if it corresponds to a sequence of distinct vertices $(v_1,..,v_n)$, such that $e_i = (v_i, v_{i+1})$. Two vertices u, v are *connected* if there exists a path between them, and if G is undirected, we call the sub-graph H of G a *connected component* if and only if there exists a path between every pair of vertices in H which contains a subset of the vertices in G.

The reasoning behind invoking the notion of connected components of the undirected retweet network is to, on a high level, make sure that a meaningful discourse between users, in terms of being influenced by and influencing others, exists within the connected component. In practice, we could have a very disconnected network with lots of unconnected components, which would mean that most users only interact and retweet a few selected users. Another interesting case would be if the network would have a few significantly large components; this would suggest the existence of a set of discourses, where the users in their respective component do not interact – perhaps because of political differences reflected in large "echo chambers". To find all connected components in the retweet network, the `GraphFrames` framework in Spark was used. The result showed that 6,083,687 i.e., 85.6% of the 7,111,140 users were in the same connected component. The remaining users were scattered around in smaller connected components, with the largest being 74 users. These users were therefore discarded from further analysis.

4.2.3 Community Detection

While the data set contains tweets using the hashtags of the counter movements #AllLivesMatter and #BlueLivesMatter, in practice, users associated with these movement did not necessarily use these hashtags, but often used the hashtag #BlackLivesMatter either ironically or to get more attention. Thus, just using simple querying on the hashtags in the data set, did not suffice to get a sample of users from these movements. To get a better sense of the relationship between users, we instead therefore used the community detection algorithm known as *Label propagation algorithm (LPA)*. LPA is a semi-supervised machine learning algorithm, which seeks to assign labels to nodes in a network, where each label maps to a specific community inside the network (Raghavan et al., 2007). In Spark's GraphX framework, the algorithm is implemented using Pregel API (Malewicz et al., 2010), which allows for parallel computation when processing graphs. On a high level, Pregel computations are a sequence of iterations, defined as *supersteps*, where for every superstep, each vertex in the graph runs a user defined function. This local vertex-centric approach where each vertex is processed independently in parallel, in contrast to the more classical iterative graph algorithms where each vertex is visited one by one, naturally induces distributed implementations that can computationally scale to arbitrarily large networks. In distributed LPA, implemented as a Pregel program , each vertex in the graph is initially assigned its own distinct vertex label to represent its initial community label. At every superstep, vertices send their community label to all out-neighbours and update their label to be the mode community label of incoming messages from their in-neighbours. Although the algorithm can have trivial or oscillating solutions without guarantees on convergence, it works well in practice on real data as we found by running LPA on the largest connected component with 10 supersteps and investigating at least the most influential set of users within each community manually.

4.2.4 Exploring Ideological Diversity

By looking at the twenty most retweeted users, we see that eighteen of these fall into the same pro-BLM community, with 155,229 users. Andy Ngo is in a community with 26,624 users. This is interesting when we remind ourselves from Table 3 that he is the sixth most retweeted user, and if we assume that most of his retweets come from his relatively small community, it suggests that he has a very loyal set of core followers. The questions that arises then are if we can identify this core set of followers, and moreover if we also can identify a similar core followings in the pro-BLM community. In the same community where we find Andy Ngo, we also have prominent conservative commentators such as Candance Owens, Glenn Beck, Steven Crowder, Paul Joseph Watson, Dave Rubin, and also Republican senator Ted Cruz, and Raheem Kassam from the Reform UK-party (formerly known as The Brexit-Party), along with others. It is worth mentioning that all of the twenty most retweeted users in this community are users with largest followings (over 25,000 followers). Thus, the phenomena of users with small followings reaching a larger audience does not exist to the same extent in this community when compared to the pro-BLM community.

The last of the twenty most retweeted users is the official account of the South Korean pop (K-pop) group BTS, who has their own community. The communities for the top ten most retweeted users are presented in Table 3 and a sample of tweets from the pro-BLM and anti-BLM communities are presented in Table 4.

Note how the textual content of the tweets from the two communities differ. By going through the label propagation algorithm we seem to have identified the two different political camps. Moreover, we note that usage of the hashtag #BlackLivesMatter is prominent in the anti-BLM community. Thus, we can conclude that just filtering by the anti-BLM #AllLivesMatter and #BlueLivesMatter would not have sufficed to identify these communities.

Thus, through the use of (1) retweet network, which encodes retweets, one of the clearest signals of directional ideological concurrence of the retweeter with the tweeter, (2) distributed label propagation on such a retweet network to detect communities of users who are in ideological concurrence within each community, and finally (3) listing the top K most retweeted tweets within each such community, we have a simple yet effective mechanism to explore the *ideological diversity* that is representative of the communities, independent of their sizes and activity levels, i.e., the number of users and intensity of interactions in Twitter. We found this simple three-step process to be an effective approach to identifying the pro/anti-BLM tweets before further analysis.

5 JOINT MEDIA MODELING

In this Section we examined the interplay between the Twitter and GDELT data sets by looking at the Granger causality between them. For this we proposed simple two-dimensional Hawkes processes with an exponential kernel. The timeline for this joint modeling was three days after the death of George Floyd over the 24-hours-long period between midnight of 28 May and midnight of 29 May, which is when the protests had just started to spread nationwide across the US, and also become violent.

5.1 Model and Data

In dimension one we had the Twitter data. To control the magnitude of the data we only considered original tweets, i.e. all retweets were filtered out, that had at least one retweet, to filter out tweets made by users with a negligible following. Moreover, we examined the 20 largest communities and identified one anti-BLM (the same community identified in the previous section), and filtered out all tweets made by users from that community, so that we only considered pro-BLM tweets. This left us with $10,774$ tweets.

In the second dimension we had records from the GKG-database from GDELT. The records were first filtered on mentioned themes, and only those reporting events of protests were selected. This naturally lead to some noise in the data, due to not being able to precisely filter out only the events mentioning protests relating to the Black Lives Matter-movement. To reduce this noise, we also filtered on records that mentioned George Floyd. While in theory a record could report a BLM related protest without mentioning George Floyd, we reasoned that since our timeline of interest was three days after his passing, most records should mention George Floyd to give the reader some context for the reported protest. To handle that the GKG-database updates in intervals every 15 minutes, every record got a randomised timestamp in the fifteen minute interval prior to it being added into the database, to get the records in continuous time. With this query in the selected time interval, $3,341$ records were found.

Given this data, we jointly model events in social and mass media by fitting the multivariate Hawkes process in Definition 2.3. We want to test whether or not Granger causation exists between dimensions 1 and 2 representing events in Twitter and events in mass media from the GDELT project, respectively. As per Theorem 2.1, parameter $\alpha_{12} = 0$ if and only if mass media events do not Granger cause Twitter events, and vice versa for $\alpha_{21} = 0$.

Table 4: Sample tweets from the pro-BLM and anti-BLM communities.

Pro-BLM community
i can't stand by and continue to live in a world where the color of your skin is an automatic target on my family, friends, and neighbors backs. tri-city we must come together to support our communities. THIS. IS. AMERICA. BE THE CHANGE YOU WANT TO SEE. #blacklivesmatter https://t.co/XIDSNqgx6Q
Thread of people who took it upon themselves to trivialise the current situation going on and #BlackLivesMatter
#BlackLivesMatter Houston is hosting a protest march this FRIDAY at 2PM starting at Discovery Green demanding justice for #GeorgeFloyd White allies, y'all gotta do better and this is a place to start. Everyone who's able should be there. https://t.co /EbWeBrZneP
Aiyana Jones a 7 YEAR OLD CHILD who was shot in the head by an officer, when the officer raided the wrong house. A 7 year old girl didn't deserve to be killed because of disgusting reckless officers. Acab and BLM, never forget this girls name! #BlackLivesMatter https://t.co/HCWzabkFv4
So protest in Huntsville, TX was small, but that was no surprise. We're a small town and most things just caught up to the present on the outside...at the end of the protest on my way home, I saw something I never noticed. This is why we do what we do. #BlackLivesMatter https://t.co/gTuCilB7mi
Anti-BLM community
Black people are 80 times more likely to kill white people in England/Wales than the reverse! And yet, #BlackLivesMatter more than others? EXPLAIN... Check the stats: https://t.co/DmPDVVGbSo https://t.co/qxXmuNIh2X
#BlackLivesMatter should now be classified as an extreme political hate group.. Simple.. https://t.co/mFh56qCpo9
#DontTakeTheKnee #DontTakeTheKnee please get this trending Sick & tired of the #ScumMedia telling us what we should do! Well I say #DontTakeTheKnee #BLM is a terrorist organisation. Do your homework! #AllLivesMatter #WhiteLivesMatter #ISTANDwithDominic Raab @SkyNews
Then someone gets stabbed and they want the police back after running them out of town. Ha you couldn't make it up #BlackLivesMatter #blm #thugs #brixton https://t.co/1uVXQ63UT2
Just saw a video of #BlackLivesMatter protest in #Reading - looks like 3 white people have been stabbed and in a bad way! Now if this turns out to be a race attack, I'm going to blame the #Media. They've been stoking up tensions between blacks and whites for weeks now!

5.2 Results

The data was fitted using `python` library `tick` [9]. `tick` requires that the decay parameters β_{ij} are given as constants beforehand, which then allows highly efficient fitting of the remaining parameters μ_i and α_{ij}, using accelerated gradient descent (Bacry and Muzy, 2016). The problem of fitting the decay parameter β in the exponential kernel is well-known (Santos et al., 2021), and is due to the fact that while the baseline parameter μ and excitation parameter α can be efficiently computed via convex optimisation, this is not always true for β. With this in mind, we proposed three different models where the decay parameters β_{ij} were handled differently:

- $\mathcal{M}_0$: $\beta_{ij} = 1,\ \forall(i,j) \in \{1,2\} \times \{1,2\} =: \{1,2\}^2$
- $\mathcal{M}_1$: $\beta_{ij} = \beta \in (0,\infty),\ \forall(i,j) \in \{1,2\}^2$
- $\mathcal{M}_2$: $\beta_{ij} \in (0,\infty),\ \forall(i,j) \in \{1,2\}^2$

To compare the different models, we looked at (i) the *Akaike information criterion* $\text{AIC} = 2k - 2\ln(\hat{L})$, where k is the number of estimated parameters, and $\hat{L}$ is the maximum likelihood of the model, (ii) the *relative likelihood* $\exp((\text{AIC}_p - \text{AIC}_q)/2)$, where the AIC values for models p and q satisfy $\text{AIC}_p < \text{AIC}_q$, and (iii) the likelihood-ratio test statistic $\lambda_{\text{LR}} = -2\ln(\hat{L}_p/\hat{L}_q)$.

5.2.1 Comparison Between $\mathcal{M}_0$ and $\mathcal{M}_1$

Setting $\beta_{ij} = 1$ for all i, j in model $\mathcal{M}_0$ gave us the log-likelihood value of 372.981, and AIC $= -733.963$ (where $k = 6$ for the two estimated baseline parameters μ_i and the four excitation parameters α_{ij}. For model $\mathcal{M}_1$, we did a sequential grid-search over β's, by using the convex optimiser in `ticks` to quickly obtain the most likely μ_i and $\alpha_{i,j}$'s for each fixed $\beta_{i,j} = \beta$, to find the most likely parameter $\hat{\beta} = 6.17$, with the maximum log-likelihood value of 384.771 and AIC $= -755.542$ (where $k = 7$ since we now also estimate β).

The relative likelihood of the models was 2.0624×10^{-5}, i.e., model $\mathcal{M}_0$ was 2.0624×10^{-5} times as probable as model $\mathcal{M}_1$ to minimize the information loss. Since $\mathcal{M}_0$ is nested in $\mathcal{M}_1$, i.e., the parameter space of $\mathcal{M}_0$ is a proper subset of that of $\mathcal{M}_1$, we do a likelihood ratio test and reject $\mathcal{M}_0$ in favour of $\mathcal{M}_1$ ($\lambda_{\text{LR}} = 23.5781, \text{p-value} < 10^{-7}$).

5.2.2 Comparison Between $\mathcal{M}_1$ and $\mathcal{M}_2$

Model $\mathcal{M}_1$ and $\mathcal{M}_0$ assume that the decay parameters β_{ij}'s are identically $\beta \in (0,\infty)$, i.e., the decay parameter within each dimension and between every pair of dimensions is given by the same value. The real-world interpretation of this is that tweets and mass media reports stay relevant for the same amount of time into the future, which seems like a major assumption as mass media dissemination and social media communication are fundamentally different in nature. To account for this, we introduced model $\mathcal{M}_2$, where each β_{ij} can vary freely in $(0,\infty)$.

We did a sequential grid search over the 4-simplex, similar to the one-dimensional case of $\mathcal{M}_1$. We found the most likely values to be $\hat{\beta}_{11} = \hat{\beta}_{22} =$

[9] https://x-datainitiative.github.io/tick/

16.170, $\hat{\beta}_{12} = 3.702$, and $\hat{\beta}_{21} = 8.638$, at the maximum log-likelihood value of 384.772, with $k = 10$ and AIC $= -749.544$. Note that despite having three additional parameters, the maximum log-likelihood of $\mathcal{M}_2$ is close to that of $\mathcal{M}_1$, with the relative likelihood of the models, likelihood-ratio test statistic, and p-value being 0.04984, 0.002121, and 0.9971, respectively. We therefore do not reject $\mathcal{M}_1$ in favour of $\mathcal{M}_2$ and choose $\mathcal{M}_1$ for further analysis.

5.2.3 Fitting the Data Using $\mathcal{M}_1$

To find whether Granger causality between the two dimensions exists, we were interested in whether parameters $\hat{\alpha}_{12}, \hat{\alpha}_{21}$ are equal to 0 or not. Fitting the data using model $\mathcal{M}_1$ with estimated decay parameter $\hat{\beta} = 6.1700$ gave us the following estimated parameters $\hat{\mu}_1 = 1.000$, $\hat{\mu}_2 = 0.998$, $\hat{\alpha}_{11} = 0.986$, $\hat{\alpha}_{12} = 0.0327$, $\hat{\alpha}_{21} = 0.0216$, $\hat{\alpha}_{22} = 0.921$. Note that the point estimates satisfying: $\hat{\alpha}_{12} > \hat{\alpha}_{21} > 0$, implies that there exists Granger causality between reported protests and tweets regarding the BLM-movement, provided we account for the errors in their estimation, i.e., their confidence intervals. We address this next using non-parametric bootstraps.

5.2.4 Hypothesis Testing

The following null hypotheses were proposed:

- $H_{0,12} : \alpha_{12} = 0$, i.e., reports of protests in mass media do not Granger-cause communication events in Twitter related to the BLM-movement.
- $H_{0,21} : \alpha_{21} = 0$, i.e., communication events in Twitter related to the BLM-movement do not Granger-cause reports of protests in mass media.
- $H_0 : \alpha_{12} = \alpha_{21} = 0$.

To get the confidence intervals for α_{12}, α_{21} we did a non-parametric bootstrap by sampling the observed data with replacement, and then estimating the parameters on the bootstrapped data under model $\mathcal{M}_1$. This was repeated 1000 times.

For α_{12}, i.e., the influence of mass media on Twitter, the 99-th percentile bootstrapped confidence interval is $(0.000, 0.09405)$, and therefore we cannot reject the null hypothesis $H_{0,12}$ that $\alpha_{12} = 0$ by the Wald test. Thus, the reports of street protests in mass media do not Granger-cause the pro-BLM interactions in Twitter.

On the other hand, the 99-th percentile bootstrap confidence interval for the parameter α_{21} that models Twitter's influence on mass media is $(0.01479, 0.02949)$, and therefore we reject the null hypothesis $H_{0,21}$ that $\alpha_{21} = 0$ by the Wald test. Thus, the pro-BLM interactions in Twitter Granger-cause the reports of street protests in mass media. We therefore also reject the common null hypothesis that there is no Granger causality whatsoever between social and mass media events around the BLM-movement, i.e., $H_0 : \alpha_{12} = \alpha_{21} = 0$.

To estimate type I error, i.e., the probability of rejecting the null hypothesis H_0, when it is true, we simulated data from the null hypothesis H_0, i.e., from the most likely parameters in $\mathcal{M}_1$, while restricting $\alpha_{12} = \alpha_{21} = 0$. For each such simulated data, we then performed the Wald test using non-parametric bootstraps by sampling the data with replacement $1,000$ times. Only one out of 100 such simulations from H_0 was rejected giving 0.01 as the Monte Carlo estimate of the Type I error.

6 CONCLUSION

We jointly model and test hypotheses about causal relationships between interactions in social media and the reports in mass media during the Black Lives Matter (BLM) protests following the death of George Floyd, by implementing open-source pipelines through `MEP` and `SPARK-GDELT` to process the data, i.e., extract, load, transform, explore, from scratch and at scale, on cloud infrastructure, and by employing self-exciting Hawkes processes and their Granger causal inference machinery.

We reject the null hypothesis that there is no causal relationship, and show that communication events in Twitter, surrounding tweets that supported the BLM movement, Granger-caused the reports of street protests in mass media from the GDELT project. However, we cannot show that the reporting of street protests in mass media Granger-caused the corresponding communication events in Twitter. We identified such pro-BLM tweets thorough a network analysis of the Twitter data to identify communities of users who have a shared ideology among an ideologically diverse set of communities.

We thus establish a verifiable causal relationship between social media interactions in Twitter that are supportive of the global BLM social movement on one hand, and global mass media reports of street protests in solidarity with the movement on the other. This suggests that activists have harnessed social media to raise awareness and mobilise street protests.

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